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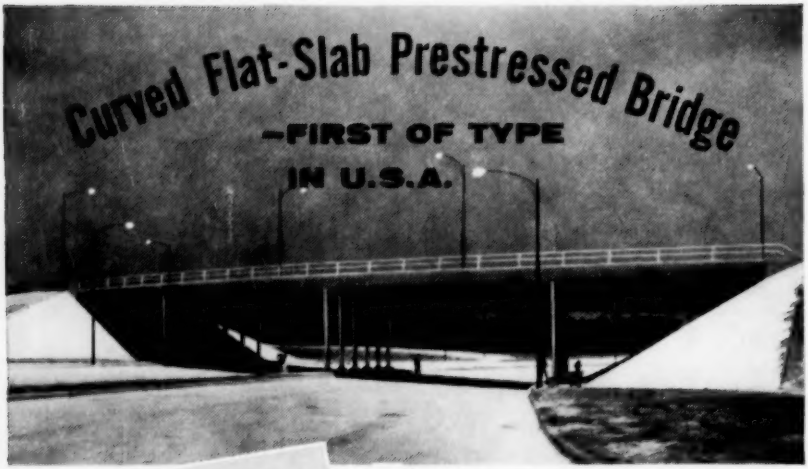

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ATTENTION . . .

A change in publication of

DISCUSSION

For papers appearing in July through December of 1956, discussions will appear at the end of June 1957 JOURNAL.

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To reduce the delay between publication of a paper and the appearance of discussion of that paper, the Technical Activities Committee has made this change. Discussion of the remaining papers in Proceedings Volume 53—January through June, 1957—will appear in Part 2, December 1957, along with the volume index as usual.

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JULY–DECEMBER 1956 DISCUSSION WILL APPEAR IN JUNE 1957

the ACI JOURNAL

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The ACI JOURNAL technical pages are a medium for reporting what is of current interest in the concrete field; in research, design, construction, maintenance, or manufacturing. Members and readers are urged to "have their say" as a means of sharing knowledge and experience; time and changing interests demand that new names and new ideas take their places beside the old. Contributions describing new materials and techniques or new applications of familiar practices are welcomed by the Institute.

To guide the flow of material the Board of Direction has assigned to the Technical Activities Committee the selection of papers, committee reports, discussions, and other contributions for JOURNAL publication. To assist in their technical appraisal a corps of experts in various segments of the field volunteer their services. From the technical standpoint, acceptable contributions present original material for improvement of design, manufacturing, or construction; confirm, revise, or upset established ideas or practices; or review, digest, or arrange accumulated experience for more ready use.

General considerations by TAC are whether the contribution will be of interest and service, will fit into a well-balanced publications schedule, and can be published at reasonable expense to the Institute.

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Title No. 53-24

Considerations in the Selection of Slab Dimensions*

Reported by Subcommittee II, ACI Committee 325

A. T. GOLDBECK
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SYNOPSIS

The principal dimensions of a pavement slab are length, width, and thickness. These dimensions and the factors which determine them are discussed separately and typical examples are given of the determination of slab thickness for specific sets of conditions.

SLAB LENGTH

Slab length is the distance between adjacent transverse joints of any kind. Determination of the proper length of concrete pavement slabs involves a consideration of several factors, some of which depend upon longitudinal slab stresses, however produced, and others upon the pavement design, construction, maintenance, and past performance under traffic. All of the factors involved must be so balanced against each other that the slab length finally chosen will result in optimum pavement performance. The economic phase of the problem is important and optimum performance must be attained at a cost which, although not necessarily the minimum, does not, however, exceed the value of the benefits produced.

The width of joint opening at a time of low temperature will be influenced by slab length; the longer the slab the greater will be the joint opening. This must be taken into consideration by the designer, for the difficulty in maintaining a joint when there is excessive movement may make it necessary to impose a limit on the slab length.

Similarly, the methods for designing longitudinal steel for concrete pavement slabs require an increase in weight of steel reinforcement as the slab length is increased. Thus, the slab length of a reinforced concrete pavement is influenced by the amount of longitudinal steel. This interrelationship will be discussed in connection with the subject of steel reinforcement.

*Prepared as a part of the work of ACI Committee 325, Structural Design of Concrete Pavements for Highways and Airports. The report was submitted to the main committee (17 members) with 15 voting affirmatively and two negatively.

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Because of the significant influence of longitudinal steel it is appropriate to discuss the subject of slab length under two headings, namely, plain concrete pavements and reinforced concrete pavements.

Plain concrete pavements

Concrete pavements built without distributed steel and without transverse joints other than construction joints placed at the end of each day of work, tend to crack into slabs of varying length. The cracks which form are caused by stresses which have several origins.

a. Frictional stresses—First, there is the direct tensile stress caused by the contraction or shrinkage of the slab when the temperature drops or the moisture content of the concrete is lowered. Most pavement concrete is placed on warm summer days and the moisture content of the slab is high at the time the concrete hardens. Lowered temperatures are likely to be encountered during the first night after placement and moisture content will begin to drop through evaporation and absorption by the subgrade. Both these changes cause shrinkage of the concrete when it has acquired but little strength. This shrinkage is resisted by the subgrade and the resulting tension which develops in the slab may cause transverse cracks to form. The spacing of these cracks will depend on the subgrade coefficient of friction and on the strength of the concrete at the time the shrinkage occurs. It may be expressed mathematically as:

$$L = 2\frac{f_t'}{F} \qquad (1)$$

in which L = length of slab, ft
f_t' = tensile strength of concrete at time crack forms, psi
F = coefficient of friction between slab and subgrade

This equation is for concrete weighing 144 lb per cu ft. Because subgrade friction is difficult to evaluate and the strength of the concrete is changing rapidly, the actual lengths of slabs which form vary considerably. Stresses having other origins will cause more frequent cracking and shorter slabs than would be produced by this contraction effect alone.

b. Warping stresses—Flexural stresses within the slab which are caused by differences in the volume change of the concrete in the top and bottom of the slab are commonly called "warping" stresses, and sometimes "curling" stresses.

Warping stresses are flexural stresses produced by the dead weight of the slab when it tends to be warped out of contact with the subgrade. There generally exists a temperature differential between the top and bottom of the slab, a higher temperature at the top than at the bottom during the day and the reverse at night. Just what that differential might be is governed by the local weather conditions. Typical differentials determined for the vicinity of Washington, D. C., are +2 to 3 F during the day and −1 deg at night, per inch of thickness. Thus, if the pavement is 8 in. thick, there

could be approximately 16 to 24 deg higher temperature at the top than at the bottom of the slab, which tends to cause the pavement to become convex upward, thus producing tension at the bottom. At night the effect is reversed; the pavement tends to warp upward at the sides and at transverse joints, and tension is thus produced at the top surface and particularly at the corners due to the warping of the slab and combines with tensile stresses produced by the wheel loads. Moisture differentials have the same general stress-producing effect although probably not as severe as that due to temperature.

Quite obviously the harder the subgrade the more severe will be the warping stresses because a hard subgrade does not yield as readily as a softer subgrade as the slab tends to become warped. The subgrade reaction pressures are more concentrated when the subgrade is hard and a relatively higher bending moment is thereby produced.

Formulas have been written to indicate the magnitude of these stresses for conditions over-simplified to such a degree that rarely do the actual conditions approach those assumed. For that reason the formulas for warping stresses have little or no practical application. This does not mean that warping stresses do not occur or do not at times develop considerable magnitude. But with the conditions under which a concrete pavement must serve, including fluctuating air temperatures, surface alternately exposed to or protected from direct sun's rays, slab in contact with a moist subgrade below it and with the top exposed to moisture conditions ranging from rainfall to winds of low humidity, the simple conditions required for warping stress formulas to be applicable do not exist. However, formulas for warping stresses serve a useful purpose in determining "order of magnitude" stresses due to warping of the slabs.

This subject of warping stresses is complicated and has been treated in theory. Apparently, warping stresses can at times be extremely high and their effect on the transverse cracking of a pavement slab is marked. Kelley* has discussed this matter and has computed warping stresses for 6- and 9-in. slabs for two different subgrades (k = 100 and k = 300) and has made certain observations which are important. These are as follows:

1. Increases in the length of the slab beyond about 18 ft in the case of the 6-in. slab, and about 24 ft in the case of the 9-in. slab, have no great influence on maximum edge or interior temperature-warping stresses. Below these limits, decreases in slab length result in rapid reduction in these stresses.

2. For the longer slabs the maximum temperature-warping stresses in the 9-in. slab exceed those in the 6-in. slab by 40 to 50 percent. However, for slab lengths less than about 17 ft for k = 100 and 13 ft for k = 300, these stresses in the 6-in. slab exceed those in the 9-in. slab by as much as 50 psi.

3. Variations in the value of the subgrade modulus have no significant influence on the temperature-warping stresses in long slabs. However, for short slabs increases in the value of the subgrade modulus result in considerable increases in the computed stresses.

*Kelley, E. F., "Application of the Results of Research to the Structural Design of Pavements," ACI JOURNAL, June 1939, *Proc.* V. 35, pp. 437-464.

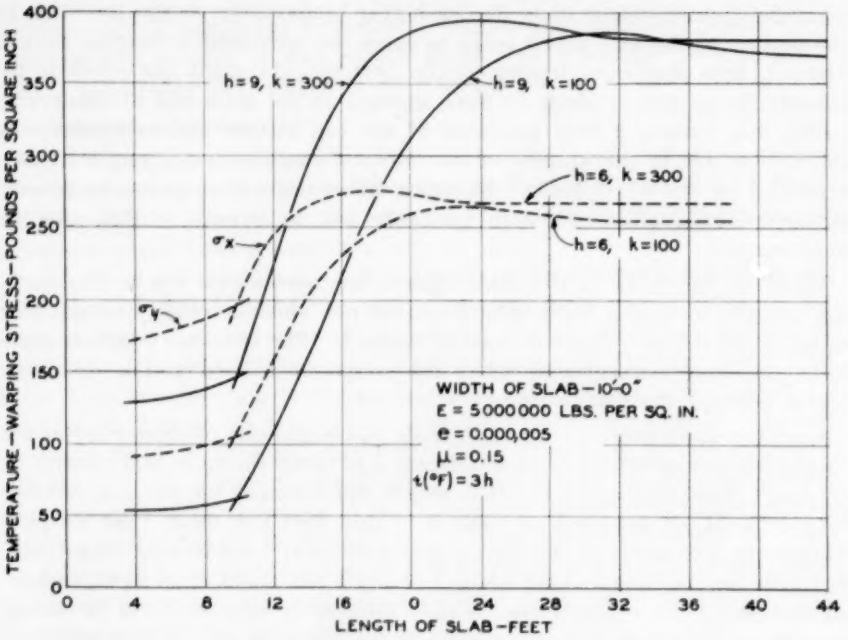

Fig. 1—Temperature-warping stresses—interior of slab

Fig. 1 and 2 show that the order of magnitude of warping stresses in the 9-in. slab for $k = 300$ may exceed those for $k = 100$ by more than 100 psi.

Warping stresses due to moisture differential are relatively small and may safely be neglected in any computations of pavement thickness. It is impracticable to show the development of formulas for warping stresses, but the reader is referred to the selected references (p. 454) and to the formulas useful in slab design appearing later in this report.

c. Other influences on slab length—The number and complex nature of the stresses which lead to transverse cracking are such that theoretical relationships alone are not sufficient to determine optimum slab length for unreinforced pavements. Supplementary information based on observations of pavements in service is needed. Pavements subjected to different conditions and made with different materials display cracking intervals not explained by established relationships.

Among the major factors affecting crack interval are the volume-change characteristics and tensile strength of the concrete, which are functions of the aggregates, proportions, and the cement used. Combinations which produce high volume changes and low flexural strengths will result, for similar conditions, in shorter slabs than more favorable combinations.

Among the principal factors which contribute to high volume changes are: (a) aggregates of high thermal coefficients of expansion; (b) high ratios of fine to coarse aggregate; (c) high water content and, to a minor degree, (d)

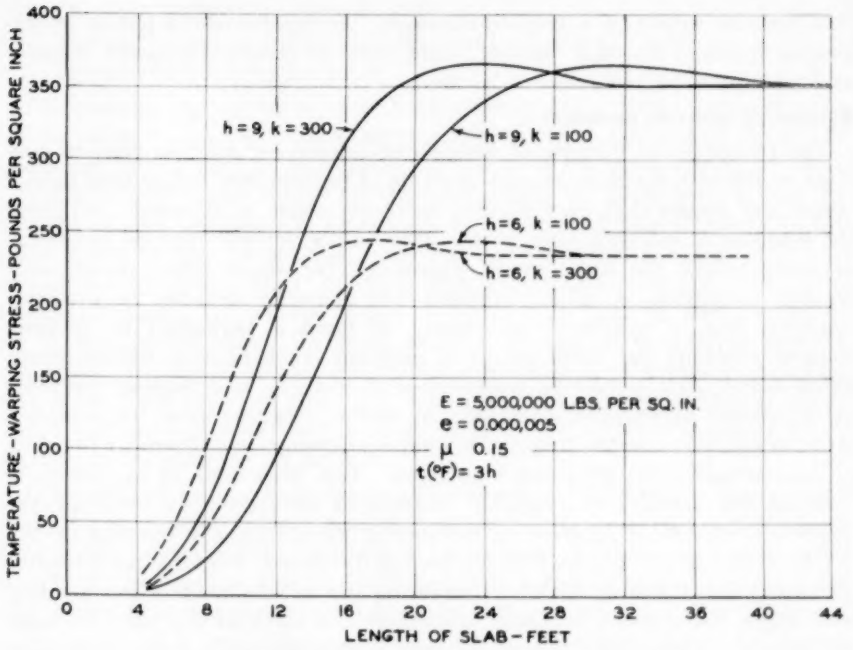

Fig. 2—Temperature-warping stresses—edge of slab

high cement factors. Factors contributing importantly to low flexural strengths are: (a) coarse aggregates containing a significant number of particles of low strength; (b) certain hard, dense, coarse aggregates with smooth surfaces which result in poor bond between the mortar and the coarse aggregate; and (c) a weak mortar, which may result from any of several causes including the strength-producing properties of the fine aggregate, the amount of cement and mixing water, and the strength-producing properties of the cement.

Characteristics causing high volume changes may coincide with those resulting in high flexural strengths, and the reverse may be true. Other factors such as underlying soil, prepared subgrade, climate, and traffic are important in their influence on cracking. To control the frequency of cracking it is good practice to build joints into the pavement. For average conditions in nonreinforced pavements 20-ft slab lengths are commonly used. Under adverse conditions it may be necessary to use a slab length of 15 ft to avoid intermediate cracking whereas under more favorable conditions slab lengths of 25 ft may be used.

In plain concrete pavements, by the use of a proper slab length, intermediate cracking will be controlled and the amount of opening of the joints will be minimized, thus making easier effective sealing. Under large volumes of heavy truck traffic load transfer devices are necessary to supplement the

load transfer action of aggregate interlock. If dependence is placed on aggregate interlock for load transfer, there must be excellent subgrade support and no expansion joints should be used.

Reinforced concrete pavements

The fundamental purpose of distributed reinforcing steel as used in concrete pavements is not to prevent cracking of the concrete but to hold tightly closed any cracks that may develop from any cause whatsoever. Although the concrete of either a plain or a reinforced section will rupture in tension at substantially the same stress, experience has shown that, under comparable conditions, a plain pavement will generally develop more visible cracking than a reinforced pavement. If bond is permitted to develop, thereby checking the development of incipient crack fissures, reinforcement holds many such cracks to microscopic dimensions and thereby prevents or delays the appearance of eye-visible cracks. Furthermore, by rendering structurally harmless the first cracks that do develop, reinforcement prevents or lessens additional progressive cracking. But, regardless of any effect in limiting the amount of cracking, distributed reinforcement performs the important structural function of preventing the progressive opening of any cracks that occur. By holding cracks tightly closed, reinforcement renders all cracks self-doweling, thereby preventing the development of independent slab edges, maintaining evenness of surface, and safeguarding the structural strength of the pavement as a whole, even though the original slabs may have become subdivided by cracking into units of any possible size or shape. Thus reinforcement serves both to reduce the amount of ultimate cracking and also to control the character of the cracks that do occur.

Distributed reinforcement, as commonly employed in pavements, is not utilized in amounts sufficient to produce a reinforced beam section such as would be used in a building or a bridge. To do so would necessitate the use of such large quantities of steel as to render a concrete pavement prohibitive in cost. Furthermore, experience has indicated that such large amounts of steel are not necessary in pavement slabs.

In recent years, for continuous reinforced concrete pavements, the theory has been advanced that joints may be completely eliminated (or practically so) and that, except for construction joints which are so completely tied that they do not function as a joint, the slab length may be increased indefinitely up to the length of the project being paved. Formulas previously used for design of distributed steel are not applicable to continuously-reinforced slabs and a few experimental projects have been undertaken to determine the practicability of the theory and how much steel would be required. It cannot be said that these experimental projects have as yet supplied a definite answer to either question. Therefore, at the present time, it is considered impractical to prepare standards of design for continuously-reinforced pavements and in this discussion of slab length no attempt will be made to cover that subject.

Since effective crack control can be maintained by the addition of distributed steel, the length of a reinforced concrete slab depends upon:

a. The cost of distributed steel
b. The initial cost of joints, including load transfer devices
c. The cost of joint maintenance

The amount of steel required to keep cracks tightly closed increases with the slab length. As slab length decreases, the number and cost of transverse joints per square yard of pavement increases. If the slabs are longer than about 25 ft (and under some conditions when they are 20 ft or even less) the amount of joint opening will be enough to reduce aggregate interlock to the point where it may be of little value for load transference. For such conditions the joints should be equipped with adequate mechanical load transference devices. Within the limits of permissible opening and closure of joints, slab length should be fixed so that the cost of distributed steel in place plus the cost of joints plus joint maintenance is at a minimum. This will differ from place to place and with changing costs of steel and labor. The lowest costs probably will be attained when the joint spacing lies approximately in the range of 40 to 70 ft.

It is difficult and perhaps practically impossible to maintain an adequate seal at a joint with present sealing material if the seasonal fluctuations in joint width exceed about 50 percent of the original width of the sealing reservoir.* If temperature were the only factor it would be easy to compute the slab length that would result in any given variation in joint width. Actually, however, moisture content is usually highest at low temperatures and lowest at high temperatures. Thus, the change in moisture content tends to cause the slab length to increase when the temperature causes it to decrease, and vice versa. Also, resistance of the subgrade slightly reduces the amount of movement of slab ends. However, maximum expansion of the pavement and maximum closing of expansion joints take place in the spring, due to the combined effects of moisture and temperature.

Since the economical length of slab, considering initial cost of joints and distributed steel, extends up to 70 ft and because it is more difficult and costly to maintain a joint with an opening larger than that usually attained with a 50-ft slab length, it seems possible that the economical slab length for a reinforced concrete pavement is in the neighborhood of 60 ft. Some analyses, however, indicate that a length not in excess of 40 ft might be more desirable to minimize the difficulty of joint maintenance.

Slab lengths required by state highway departments

As a result of a survey of concrete pavement practices in the various states it was found that, in 1952, slab lengths varied from 15 to 100 ft and few changes in lengths were reported in 1952 as compared with 1948. Sixteen states reported construction of 15- to 20-ft slab lengths in 1948 and also in 1952, several states with reinforcement but most of them with plain slabs. In 1948 and again in 1952, seven states reported the slabs they constructed were between 70 and 100 ft long. The only changes in slab lengths reported

*Anderson, A. A., "Expansion Joint Practice in Highway Construction," *Transactions*, ASCE, V. 114, 1949, pp. 1159-1168.

were in the 30- to 40-ft lengths. In 1948, eight states reported slab lengths of approximately 30 ft and five states reported slab lengths of approximately 40 ft, whereas in 1952 only five states reported slab lengths of 30 ft and eight states reported slabs of approximately 40 ft in length.

SLAB WIDTH

The over-all width of a pavement is dependent largely on the traffic requirements, whether in a highway or an airport. Once the over-all width has been established, the pavement should be constructed so that irregularly shaped longitudinal cracks caused by excessive transverse bending strains due to temperatures and loads will not develop. The prevention of excessive transverse stress requires the building of longitudinal joints into the slab, thus dividing the pavement into lanes with adequate transverse resisting moment.

Highway slabs

a. Pavement width requirements for traffic—One of the most important conditions which affects the capacity of a highway is lane width. Studies* made by the Bureau of Public Roads indicate that desirable clearances for commercial vehicles require a 24-ft pavement, or a 12 ft lane width. When a passenger car meets a commercial vehicle on a pavement 22 ft wide, the passenger car has the desired center and edge clearances, but the commercial vehicle does not. Hazardous traffic conditions exist on pavements less than 22 ft wide that carry even moderate volumes of mixed traffic. Narrow lanes have a considerably lower capacity than the 12-ft lanes which are at present considered necessary for heavy volumes of mixed traffic. The practical capacity of a 9-ft lane is only about 70 percent of that of a 12-ft lane. However, on secondary roads, 11-ft, and even 10-ft lanes are sometimes used, but are not recommended for pavements carrying the heaviest vehicles. The determination of pavement widths for channelized intersections and also of minimum desirable widths of turning lanes are special problems in width design. Of primary importance for determining lane width in such cases are radius of curvature and design and length of vehicle. This subject is too complicated for adequate treatment in this report and is available elsewhere.†

b. Slab width requirements to prevent longitudinal cracking—Analysis of and experience with highway pavement slabs above 12 ft wide shows that excessive transverse bending stresses due to temperature and loads are apt to develop and thus produce irregularly shaped longitudinal cracks near the middle of the slab. The type of longitudinal joint, whether construction or hinge type, depends on the width of the paving lane. Some states pave two traffic lanes in one operation; others pave one lane at a time. In the former case, a longitudinal weakened plane joint is installed or produced by sawing at early age and before use, while in the latter case a construction joint is required.

*Tarragin, A., "Effect of Roadway Width on Vehicle Operation," *Public Roads*, V. 24, No. 6, 1945, pp. 143-160.
†Bellis, W. R., "Directional Channelization and Determination of Pavement Widths," *Bulletin* No. 72, Highway Research Board, 1953.

Airport slabs

Runway pavements are considerably wider than highway pavements. Early runways consisted of landing strips 500 ft wide made of packed native soil with or without turf. For the more important fields, the then Bureau of Air Commerce recommended a 100 ft width of "hard surface" in the center of the strip. This width of 100 ft represents the early thinking regarding the required width. A search of the early Bureau of Air Commerce documents does not reveal the reasoning for the choice of 100 ft. It can, therefore, be presumed that the choice was arbitrarily made from experience. As aircraft became larger, this width gradually increased so that during the second world war the acceptable standard for large fields was 150 ft. Today the very large fields have widths of 200 and up to 500 ft, 150 ft being retained for all other airports serving scheduled air transport. Taxiway widths have gradually increased from 50 ft for the moderate size airports to 100 ft for the largest fields.

Portland cement concrete runways and taxiways are normally constructed in longitudinal strips, 20 to 25 ft in width. The spacing of the longitudinal construction joints is, therefore, dependent primarily on the width of the paving equipment. Midway between the longitudinal construction joints, a weakened plane joint of the dummy type is installed. In its latest engineering manual the Corps of Engineers has recommended elimination of the center joint for all slabs 10 in. or more in thickness.

SLAB THICKNESS

The reinforcement or replacement of concrete pavements that are inadequate for vehicles or aircraft, heavier than the types for which the pavement was originally designed, is an extremely costly procedure. In military airfields, it involves the immobilization of runways and taxiways with consequent delays in the training programs. In the case of heavily trafficked highways, it involves serious problems in the re-routing of the traffic or the costly construction of miles of detour roads to be used during the reconstruction of the main thoroughfares.

Basic concepts

During the past 40 years many investigations have been made looking toward the rational determination of pavement thickness. Important facts have been disclosed as a result of these investigations; and the causes of high stresses are now well known. At this point it is well to review in a general way some of these important facts as they relate to slab thickness:

1. Warping stresses due to differential temperatures between the top and bottom of the slab are daily occurrences. Their magnitude is high and they, combined with load stresses, can and often do produce transverse cracking in the pavement.

2. Heavy wheel loads also produce high bending stress, especially along the edges of the slab and near the corners, but these are of lesser amount in the center of the slab. Excessive bending can produce accentuated pumping.

3. Load transfer across joints or cracks helps to relieve bending stresses at joints, edges, and corners.

4. Distributed steel does not prevent cracking but it does hold the slab together at the cracks so that faulting of the pavement at these cracks is avoided. When cracks occur, the high warping stress which formerly existed there becomes negligible.

5. Subgrade friction in slabs longer than suitable for unreinforced pavements should be overcome by longitudinal steel which must be sufficiently strong to prevent its failure due to excessive tension.

6. Concrete suffers fatigue under repeated heavy loading and the importance of this fact must at least be considered in any theory of pavement design.

7. Concrete in pavements generally fails by tension produced either directly or through bending.

There are other facts which are important but the above are the most critical.

Recommendations

Warping—a. If the warping stress is not combined with the load stress for determining the thickness in plain concrete pavements, the spacing of joints, as discussed elsewhere, must be kept to a small value, for otherwise the combined warping and load stress will cause a transverse crack to form, and as this crack opens, as it well may, aggregate interlock disappears and the slabs will fault, thus producing destructive impact from traffic. In plain concrete slabs with properly spaced joints, direct longitudinal tension due to subgrade friction would be too small to consider for use in thickness design.

Accordingly, when designing plain slabs, warping stresses as well as load stresses must be considered and the joints should be spaced so as to prevent intermediate cracks from forming due to these combined stresses.

b. When a slab is adequately reinforced longitudinally, although the warping stress combined with load stress is almost as apt to create transverse cracking as in the plain slab, the crack which forms will be a fine crack and the warping stress will almost disappear at the crack where it formerly was of considerable magnitude, but the aggregate interlock will persist, thus preventing the slab from faulting. Thus there seems to be no need for taking warping stresses into account when designing a reinforced concrete pavement.

Direct tension—As concrete is weaker in tension than in compression, failure will generally be produced under conditions which produce high tensile stresses, whether by flexure or by direct tension. The contraction of the pavement due to drying shrinkage or a decrease in temperature has the effect of producing direct tension caused by the restraint of the subgrade. These tensile stresses are relatively low and need not be considered for design purposes.

Fatigue and allowable stress—Concrete suffers fatigue when the stress exceeds about 55 percent of the modulus of rupture, but if the design is such that the stress is less than 55 percent of the modulus of rupture, for practical purposes there is no fatigue. When the loads are heavier and produce a higher bending stress, more rapid fatigue takes place. During the years as the number of applications of these fatiguing stresses is increasing, so is the modulus of rupture of the concrete with age. Likewise, in service there is

a period of recovery between loads which should be taken into account, for it helps to neutralize this fatiguing action. These considerations lead to the thought that if the pavement is designed so that the maximum stress is no greater than half the modulus of rupture, fatigue will not enter into the design problem as an important factor. For this reason it seems logical to so design the pavement thickness that under the assumed maximum wheel load the stress will be restricted to roughly half the modulus of rupture. The assumed wheel load should be greater than the legal load limit, and it is well to assume an additional 10 percent for impact.

Combined warping and load stresses—The warping action of differential moisture or temperature between the top and bottom of the slab may produce severe bending stresses and hence their possible combination with other bending stresses, principally those due to wheel loads, must be considered. The following combinations of loading conditions are possible:

a. Night conditions: slab generally less contracted at bottom than at top and hence curled up at corners; temperature, lower at top than at bottom. Loads along edge of slab and at corners—here there exists combined effect of load, temperature, and moisture which results in high tension at the corner of the slab at the top surface and also transverse tension at the top of the slab.

b. Daytime: high temperature and expansion at surface, but generally low moisture and shrinkage at surface. Slab somewhat arched at center. Loads have no severe stress effect at edge or corner since pavement is in contact with subgrade at edge. Some tension at bottom of slab at center due to loads and weight of slab, but transverse tension is not important when slab width is reduced by use of a longitudinal joint. Transverse tension was at one time important when full width slabs were used. Inside wheel loading is more important during the day than at night due to the upward warping of the pavement.

Undoubtedly at times the warping stresses and wheel load stresses are additive, but the frequency of the simultaneous occurrence of the highest load and warping stresses is low and need not be taken into account except in unreinforced slabs.

Load stresses—Highway pavements, in particular, are subjected to widely varying wheel loads, generally consisting of a high frequency of light and a lower frequency of heavy wheel loads. The heavy wheel loads produce high bending stresses and the light loads an almost negligible stress. Concrete suffers fatigue when stressed above 55 percent of its modulus of rupture, and the closer the bending stress approaches the modulus of rupture, the greater will be the fatiguing effect, with the result that a comparatively few heavy loads may cause failure. Naturally, then, the magnitude of the wheel load which should be used in design theory is highly important. Several attempts have been made to choose a rational wheel load for design purposes taking into account the fatigue of concrete. It is here considered that the maximum wheel load for use in design should be that which the pavement will receive at infrequent intervals.

Separate design criteria are applicable for concrete airport and highway pavements. For airport pavements computations of wheel-load stresses

for any given load and for different thicknesses of concrete refer to loading at an interior point of the slab. In the case of concrete highway pavements the computed stresses refer to corner slab loading.

Concrete properties and tests—Since the theory of concrete pavement thickness design has reference to the flexural strength of concrete, tests of concrete beams in flexure rather than the strength in compression of concrete cylinders should be included in all specifications for highway and airport pavement construction.

The factor of safety against overstressing should have reference to direct measurements of the flexural strength and not to the estimated flexural strength obtained from compressive strength data.

For satisfactory durability and low maintenance cost it is desirable to have a daily average flexural strength of 650 psi, provided aggregate and cement characteristics permit attaining this strength economically. Should 650 psi be economically difficult to attain, a requirement of 600 psi may be used provided the slab thickness is designed accordingly.

The method of making concrete specimens for tests in flexure should follow "Standard Method of Making and Curing Concrete Compression and Flexure Test Specimens in the Laboratory," ASTM C 192-49.

The method of test for strength in flexure should follow "Standard Method of Test for Flexural Strength of Concrete (Using Simple Beam with Third-Point Loading)," ASTM C 78-49, except that not less than three beams should be made from any batch of concrete to be tested.

The maximum allowable deviation from the mean in any number of tests of a given batch of concrete for strength in flexure should not exceed 10 percent, and for thickness design purposes the maximum allowable fiber stress should not be greater than 50 percent of the average modulus of rupture specified.

For the purpose of designing the thickness of pavement slabs, it is considered that distributed reinforcement does not add to the flexural strength of the concrete.

Theory for thickness design of pavements and its limitations

Perhaps the most widely quoted theory for the thickness design of pavements is that of H. M. Westergaard. Others have also made notable contributions to pavement design theory and are referred to in the selected references, p. 454.

The theory of Westergaard involves one important assumption that may be reduced to the simple expression:

$$p = -k Z \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

In Eq. (2), p is the unit reaction of the earth support acting on the slab, Z is the deflection of the slab, positive downward, and k is the modulus of foundation reaction. Obviously, Eq. (2) is not applicable to nonuniform or spot support of the slab. The theory does not pertain to the case of unsup-

ported corners and edges of slabs. The Westergaard k is assumed to be constant over the entire paved area and during the life of the pavement. Unsupported sections of slabs are developed by nonuniform compaction of the subbase or subgrade, by traffic, by volumetric changes in the subbase, by pumping at joints, and by frost action. Adequate subgrade materials, subgrade preparation, subbase design and construction and drainage must, therefore, be first realized if the rational Westergaard method of design of concrete pavement thickness is to be reliable.

Design of concrete pavement thickness

For finding the modulus of foundation reaction, k, it is considered that a satisfactory procedure is by the use of loading tests with a 30-in. diameter steel plate. This test should be made, whenever possible, on trial sections of variable thicknesses of compacted subbase or of compacted subgrade if the subgrade material is suitable for a subbase. The control of moisture and density of the subbase is of paramount importance in this connection.

The rational method of Westergaard has appeared in various publications and will not be presented here. It is recommended, however, that only the more recent publications be applied in the design of the thickness of concrete pavement (see selected references, p. 454).

Computations of slab thicknesses

Wheel loads produce flexural stresses in a concrete pavement. The pavement fails under a single application of load when the flexural stress is equal to or greater than its flexural strength or modulus of rupture. The factor of safety (F.S.) against overloading of the slab is:

$$\text{F.S.} = \frac{\text{M.R.}}{f_t'} \qquad (3)$$

In this expression, M.R. is the modulus of rupture (psi) and f_t' is the maximum allowable bending stress (psi), provided it is not greater than 50 percent of M.R.

THICKNESS DESIGN FOR AIRPORT PAVEMENTS

Two procedures, one for a load on a single tire and one for a load distributed equally on dual tires will be described.

Case I—Single tire loading

The choice of coordinate axes is arbitrary. Let y be in the longitudinal direction of the concrete highway or runway pavement and let the x direction be normal to that of the y direction. Let the corresponding wheel load normal tensile stresses be designated as $t(y)$ and $t(x)$. The tire contact area tends to be of elliptical shape, the major axis having a length of from 1.5 to 2.0 times the length of the minor axis. Let a_e and b_e denote the semi-major and semi-minor axes, respectively. Let h denote the slab thickness at an interior point and

P the load on the single tire. Then the Westergaard expression for $t(x)$ and $t(y)$ (Poisson's ratio assumed to be 0.15) when reduced to simplest form is

$$\left.\begin{matrix} t(y) \\ t(x) \end{matrix}\right] = \frac{P}{h^2}\left[0.316 \log_{10} \frac{E_c h^3}{k \dfrac{(a_e + b_e)^4}{2}} \pm (0.203) \frac{(a_e - b_e)}{(a_e + b_e)}\right] \quad \ldots\ldots\ldots\ldots (4)$$

In this expression, the minus sign on the right corresponds to $t(x)$ and the plus sign to $t(y)$.

Example—Compute $t(x)$, $t(y)$, and the factor of safety for the following conditions:

E_c	= 4,000,000 psi	k	= 200 lb per cu in.
h	= 10 in.	M.R.	= 700 psi
a_e	= 20 in. and b_e = 10 in.	P	= 50,000 lb

Substituting these values in the expression for $t(x)$ and $t(y)$, one obtains:

$$t(x) = 377 \text{ psi}$$
$$t(y) = 445 \text{ psi}$$
$$\text{Then F.S.} = \frac{700}{445} = 1.57$$

Case II—Dual tire loads

For dual tire loading, the following expressions are used, assuming that Poisson's ratio for concrete is 0.15.

$$\left.\begin{matrix} t(x) \\ t(y) \end{matrix}\right] = \frac{3P}{2\pi h^2}\left[(1.15)(B + R) \pm (0.425)(C + S)\right] \quad \ldots\ldots\ldots\ldots (5)$$

The coefficients, B, R, C, and S are "space and area coefficients" and are determined numerically by the spacing of loads and the shapes and sizes of the loaded areas. For elliptical tire contact areas, $C = 0$ and $B = 0.1159$.* With these values the foregoing expression may be written in the following form:

$$\left.\begin{matrix} t(x) \\ t(y) \end{matrix}\right] = \frac{3P}{2\pi h^2}\left[(1.15)(0.1159 + R) \pm (0.425S)\right] \quad \ldots\ldots\ldots\ldots (6)$$

With reference to Fig. 3, the stresses at points (such as 1 and 2) in the concrete below load A are *within* ellipse A and *outside* ellipse A′. The point 3 is outside both ellipses. The expressions for the coefficients, R and S, are different for points inside and outside the ellipse. For example, the stress at point 2, under load A, is caused by the superposition of loads A and A′ at this point. For the stress at points such as 1 and 2 due to load A alone, the following expressions are used for computing the numerical values of R and S.

*See references 3, 4, and 5.

$$R = 0.5000 + \log_e \left[\frac{2L}{a_e + b_e}\right] - \frac{x^2}{a_e^2 + a_e b_e} - \frac{y^2}{a_e b_e + b_e^2} \quad \ldots\ldots (7)$$

$$S = \frac{b_e - a_e}{a_e + b_e} - \frac{2a_e b_e}{(a_e + b_e)^2}\left[\frac{x^2}{a_e^2} - \frac{y^2}{b_e^2}\right] \quad \ldots\ldots (8)$$

For point 1 (Fig. 3), x and y are both zero since this point is the origin. For point 2, x has a finite value but y is zero since the point is on the x axis.

The stresses in concrete at interior points of ellipse A due to the load A′, the stresses at points such as 3 which is outside both ellipses, and the stresses within ellipse A′ due to load A require for their computation the use of the following expressions for R and S.

$$R = 0.5000 + \log_e \frac{2l}{x + \sqrt{x^2 - a_e^2 + b_e^2}} - \frac{x}{x + \sqrt{x^2 - a_e^2 + b_e^2}} \quad \ldots\ldots (9)$$

$$S = \frac{b_e^2 + a_e^2}{b_e^2 - a_e^2} - \frac{4b^2 x}{\left[x + \sqrt{x^2 - a_e^2 + b_e^2}\right](b_e^2 - a_e^2)} \quad \ldots\ldots (10)$$

In the foregoing expressions for R and S, b_e and a_e are the semi-major and semi-minor axes of the ellipses, respectively; the two elliptic tire prints being identical with respect to size and shape; l is the radius of relative stiffness of the slab (see Table 1).

$$l = \sqrt[4]{\frac{E_c h^3}{12\,(1 - \mu^2)k}} \quad \ldots\ldots (11)$$

For concrete pavement, Poisson's ratio, μ, is usually taken as 0.15.

The Westergaard expressions for place and area coefficients and for stresses in the slab utilize the first few terms of infinite series and are therefore ap-

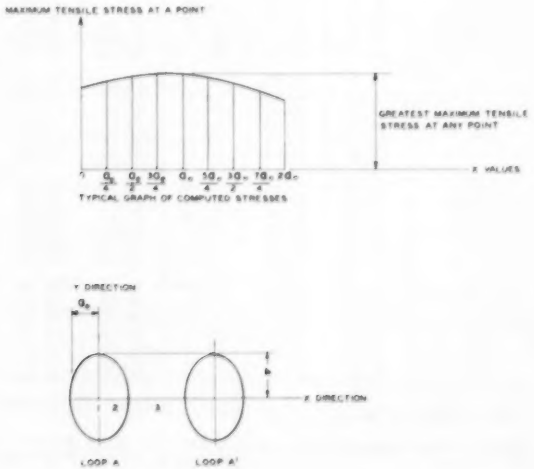

Fig. 3—Dual tire loading on concrete pavement

plicable for only "points at or near the loaded area" (Westergaard). In the present case, dual tire loading, the expressions are applicable without appreciable error provided that the distance between the ellipses does not exceed the length of the major axis, which is $2b_e$ (Fig. 3). If the two tires are farther apart than this it is much more accurate to use the influence chart and the graphical method of Pickett and Ray which is based on the exact and not the approximate procedure.

The computed stresses at points 1, 2, 3, etc., of the lower diagram of Fig. 3 are plotted in the upper diagram of this figure against the stresses at these several points. Then a smooth curve, as shown in this figure, is drawn through these plotted points. It is preferable to plot the distances of the points 1, 2, 3, etc., from the origin, in terms of a_e, the minor semi-axis of each tire footprint. The stress versus distance from the origin curve of the upper diagram has a maximum ordinate. In the diagram shown, this maximum is at a value of x that is slightly less than a_e. The position of the maximum stress necessarily changes with changes in the distance between the loaded elliptical areas, that is, the tire spacing.

TABLE 1—VALUES FOR l CORRESPONDING TO DIFFERENT k AND h*

k, lb per cu in.	50	100	150	200	250	300	350	400	500
h, in.	Radius of relative stiffness, l								
6	34.84	29.30	26.47	24.63	23.30	22.26	21.42	20.72	19.59
6.5	36.99	31.11	28.11	26.16	24.74	23.64	22.74	22.00	20.80
7	39.11	32.89	29.72	27.65	26.15	24.99	24.04	23.25	21.99
7.5	41.19	34.63	31.29	29.12	27.54	26.32	25.32	24.49	23.16
8	43.23	36.35	32.85	30.57	28.91	27.62	26.58	25.70	24.31
8.5	45.24	38.04	37.37	31.99	30.25	28.91	27.81	26.90	25.44
9	47.22	39.71	35.88	33.39	31.58	30.17	29.03	28.08	26.55
9.5	49.17	41.35	37.36	34.77	32.89	31.42	30.23	29.24	27.65
10	51.10	42.97	38.83	36.14	34.17	32.65	31.42	30.39	28.74
10.5	53.01	44.57	40.28	37.48	35.45	33.87	32.59	31.52	29.81
11	54.89	46.16	41.71	38.81	36.71	35.07	33.75	32.64	30.87
11.5	56.75	47.72	43.12	40.13	37.95	36.26	34.89	33.74	31.91
12	58.59	49.27	44.52	41.43	39.18	37.44	36.02	34.84	32.95
12.5	60.41	50.80	45.90	42.72	40.40	38.60	37.14	35.92	33.97
13	62.22	52.32	47.27	43.99	41.61	39.75	38.25	36.99	34.99
13.5	64.00	53.82	48.63	45.26	42.80	40.89	39.35	38.06	35.99
14	65.77	55.31	49.98	46.51	43.98	42.02	40.44	39.11	36.99
14.5	67.53	56.78	51.31	47.75	45.16	43.15	41.51	40.15	37.97
15	69.27	58.25	52.63	48.98	46.32	44.26	42.58	41.19	38.95
15.5	70.99	59.70	53.94	50.20	47.47	45.36	43.64	42.41	39.92
16	72.70	61.13	55.24	51.41	48.62	46.45	44.70	43.23	40.88
16.5	74.40	62.56	56.53	52.61	49.75	47.54	45.74	44.24	41.84
17	76.08	63.98	57.81	53.80	50.88	48.61	46.77	45.24	42.78
17.5	77.75	65.38	59.08	54.98	52.00	49.68	47.80	46.23	43.72
18	79.41	66.78	60.35	56.16	53.11	50.74	48.82	47.22	44.66
19	82.70	69.54	62.84	58.48	55.31	52.84	50.84	49.17	46.51
20	85.95	72.27	65.30	60.77	57.47	54.92	52.84	51.10	48.33
21	89.15	74.97	67.74	63.04	59.62	56.96	54.81	53.01	50.13
22	92.31	77.63	70.14	65.28	61.73	58.98	56.75	54.89	51.91
23	95.44	80.26	72.52	67.49	63.83	60.98	58.68	56.75	53.67
24	98.54	82.86	74.87	69.68	65.90	62.96	60.58	58.59	55.41

*E_c = 4,000,000 psi and μ = 0.15

Radius of relative stiffness $= l = \sqrt[4]{\frac{E_c(h^3)}{12(1-\mu^2)\,k}} = 24.162\sqrt[4]{\frac{h^3}{k}}$

Example—Find the maximum principal stress in the pavement at a point, $x = a$, under the following conditions:

h = 12 in.
P = load on each tire = 40,000 lb
= load A = load A′
Tire pressure = 100 psi
E_c of concrete = 4,000,000 psi
Poisson's ratio (concrete), $\mu = 0.15$
k (subgrade) = 200 lb per cu in.

Center-to-center spacing of tires = 36 in.

$$\frac{P}{1.1^* \times \text{tire pressure}} = \frac{40{,}000}{1.1 \times 100} = 364 \text{ sq in.}$$

$3.1416ab = 364$
$1.5a_e = b_e$ and $4.7124a_e^2 = 364$ sq in.
$a_e = 8.78$ in. and $b_e = 13.17$ in.
l = radius of relative stiffness = 41.43 in.

Stresses caused by load A

$$R = \tfrac{1}{2} + \text{natural log of } \frac{82.86}{8.78 + 13.17} - \frac{8.78}{8.78 + 13.17}$$

$R = \frac{1}{2} + 1.3297 - 0.400 - 1.4297$
$S = 0.200 - 0.4800 = -0.28$

Then

$$t(x) = \frac{3 \times 40{,}000}{2 \times 3.1416 \times 144}(1.15 \times 1.5456 + 0.425 \times 0.28)$$

$t(x) = 220$ psi

By load A, $t(y)$ = major stress $= \dfrac{3 \times 40{,}000}{2 \times 3.1416 \times 144}(1.15 \times 1.5456 + 0.425 \times 0.28)$

$t(y) = 251$ psi

Stresses caused by load A′

In this case, $x = 36$ in. $- 8.78$ in. $= 27.22$ in.

$$R = 0.500 + \log_e \frac{82.86}{27.22 + \sqrt{837}} - \frac{27.22}{27.22 + \sqrt{837}}$$

$R = 0.405$ and $S = -0.90$
$t(x)$ for load A′ $= 132.6\,(1.15 \times 0.521 - 0.425 \times 0.90)$
$t(x) = 29$ psi
$t(y)$ for load A′ $= 132.6\,(1.15 \times 0.521 + 0.425 \times 0.90)$
$t(y) = 131$ psi
Total $t(x) = 220 + 29 = 249$ psi
Total $t(y) = 251 + 131 = 382$ psi
For M.R. = 700 psi

$$\text{F.S.} = \frac{700}{382} = 1.83$$

If a smaller or larger factor of safety is desired, h, the slab thickness, can be decreased or increased to an extent indicated by the new computations for the altered value of h.

For airport runways subjected invariably to fast-moving loads which have a smaller stress-producing effect than static loads, it would seem that the

*The factor 1.1 is the stiffness factor of the tires as a result of which the area of contact is less than the tire load divided by the air pressure. Value 1.1 is approximate and was established by the Bureau of Public Roads many years ago.

above factor of safety is adequate. However, the designer may wish to provide for future heavier wheel loads by the use of a higher safety factor.

HIGHWAY PAVEMENTS

The relationship between slab thickness and axle loads is shown in Fig. 4 and 5 for different conditions of subgrade support. These graphs are based on Eq. (12) and (13), respectively, for protected and unprotected slab corners. The formulas developed by Pickett* are used and two conditions are considered:

a. Protected corners—Provision has been made to transfer at least 20 percent of the load from one slab corner to the other by some adequate mechanical means or by aggregate interlock. For this case

$$f_t = \frac{3.36P}{h^2}\left[1 - \frac{\sqrt{a/l}}{0.925 + 0.22a/l}\right] \quad \ldots\ldots (12)$$

b. Unprotected corners—There is no adequate provision for load transfer and one corner must carry over 80 percent of the load.

$$f_t = \frac{4.2P}{h^2}\left[1 - \frac{\sqrt{a/l}}{0.925 + 0.22a/l}\right] \quad \ldots\ldots (13)$$

In the two foregoing formulas

f_t = maximum tensile stress (psi) at the top of the slab and in the direction of the bisector of the corner angle

P = wheel load, lb (gross load plus impact). An impact allowance of 20 percent is recommended for pneumatic tires.

h = thickness (in.) of a uniform-thickness concrete slab at a corner (or equivalent thickness of a thickened-edge slab)

l = radius of relative stiffness (see Table 1)

a = radius of a circular area equal to that of the actual area of contact of a single tire with pavement. For dual tires, a is the radius of a circle of area equal to the sum of the area of contact of the two tires plus the area between them, assuming that the contact area of each tire is circular.

Example—Compute the required thickness for a dual tire load of 11,200 lb (5600 lb on each tire). The center-to-center spacing of the two tires is 13.6 in. and the tire pressure is 75 psi. The value of k is 100. The factor of safety is 2 for a modulus of rupture of 700 psi of concrete at 28 days wet curing. An impact factor equivalent to 20 percent of the load is added.

The radius, a, of the circular area equal to the area of the equivalent circle for one of the two tires plus the area of a rectangle having a length of 13.6 in. and a width equal to the diameter of the equivalent circle for one of the tires is 10.68 in. In other words, 8.65 in. is the radius of a circle having an area equal to that of a circle of radius 5.34 in., plus the area of a rectangle having a length of 13.6 in. and a width of 10.68 in.

*See references 3 and 5.

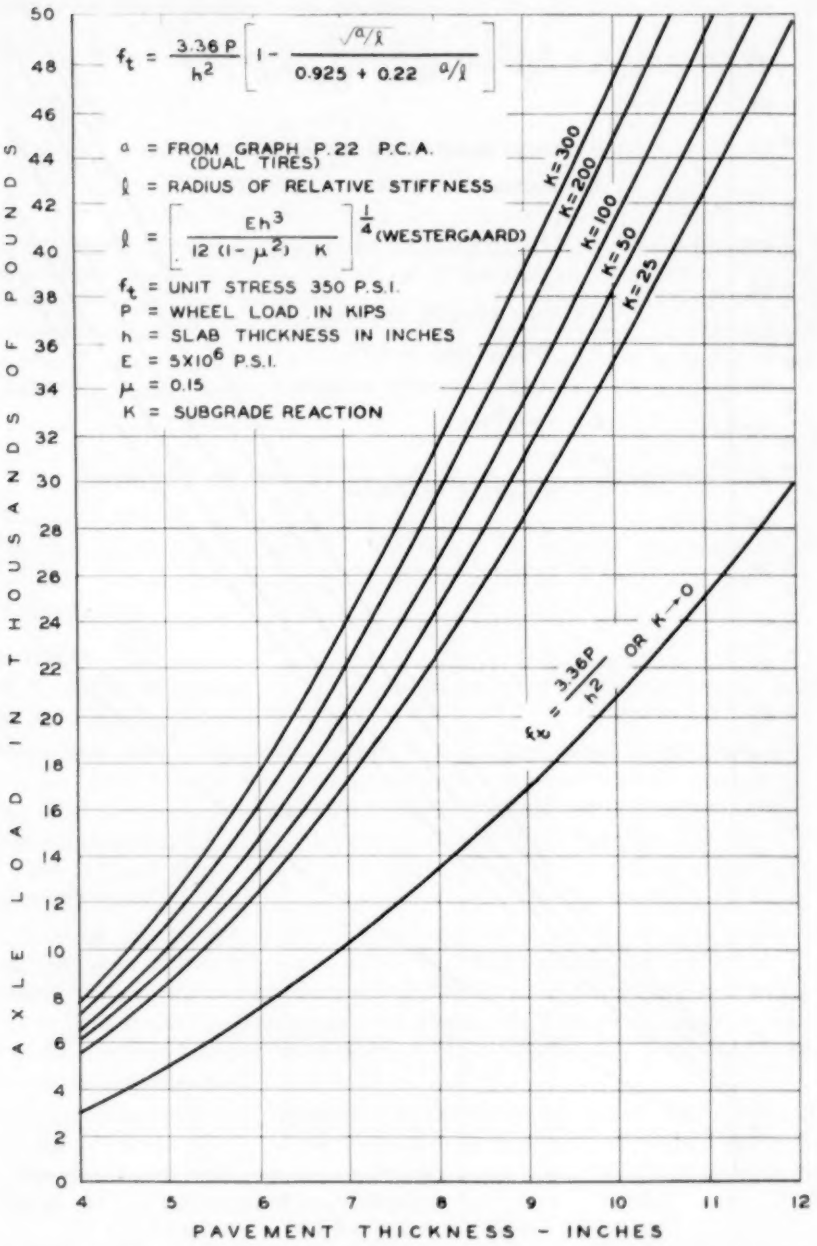
$f_t = \frac{3.36\,P}{h^2}\left[1 - \frac{\sqrt{a/\ell}}{0.925 + 0.22\ a/\ell}\right]$
a = FROM GRAPH P.22 P.C.A. (DUAL TIRES)
ℓ = RADIUS OF RELATIVE STIFFNESS
$\ell = \left[\frac{Eh^3}{12\,(1-\mu^2)\ K}\right]^{\frac{1}{4}}$ (WESTERGAARD)
f_t = UNIT STRESS 350 P.S.I.
P = WHEEL LOAD IN KIPS
h = SLAB THICKNESS IN INCHES
E = 5×10^6 P.S.I.
μ = 0.15
K = SUBGRADE REACTION
K=300
K=200
K=100
K=50
K=25
$f_t = \frac{3.36P}{h^2}$ OR K→0
AXLE LOAD IN THOUSANDS OF POUNDS
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
4 5 6 7 8 9 10 11 12
PAVEMENT THICKNESS - INCHES

Fig. 4—Design chart for portland cement concrete pavements—protected corners

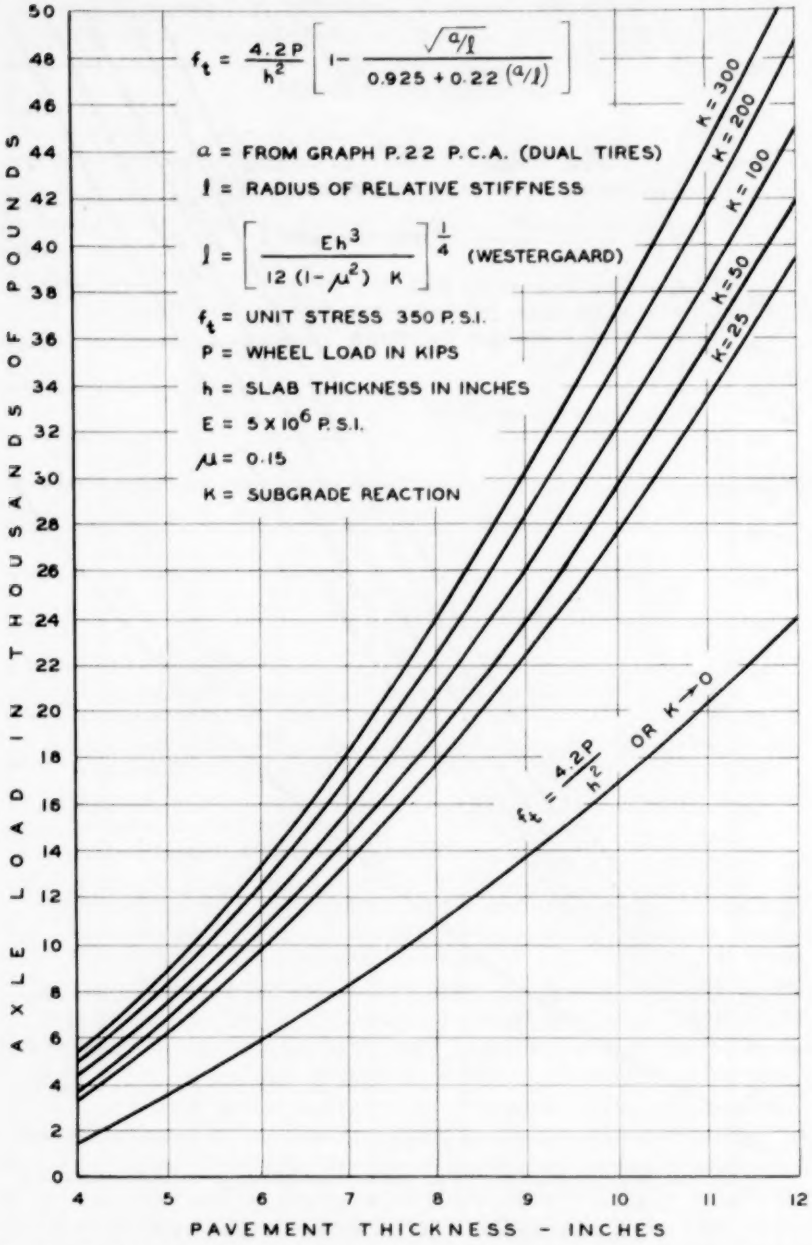

Fig. 5—Design chart for portland cement concrete pavements—unprotected corners

Using the two simplified expressions

$$h = \sqrt{\frac{3.36 \times 13{,}440\,B}{350}}, \text{ protected corners} \quad (14)$$

$$h = \sqrt{\frac{4.20 \times 13{,}440\,B}{350}}, \text{ unprotected corners} \quad (15)$$

where B is the quantity in brackets in Eq. (12) and (13), it is found by trial that h is 8 in. for protected corners and 9.5 for unprotected corners. The trial method is illustrated as follows. For the case of unprotected corners, for example, a value is arbitrarily assigned to h. Then B is computed and the expression under the square root sign in the expression on the right is computed. The procedure is repeated until the value for h and this expression are made numerically equal.

SUMMARY OF TRENDS IN HIGHWAY SLAB DIMENSIONS

Surveys conducted to determine the trends in concrete pavement design show that changes are taking place in the thinking of engineers regarding slab dimensions.

Slab length

Slab length is defined by the distance between transverse joints, both expansion and contraction joints.

Transverse expansion joints—There is a decided trend away from their use. Several years ago 27 states used expansion joints and 18 did not. [1] In 1952, only 17 states used expansion joints and 25 did not use them. In those states using expansion joints, there is a wide range in spacing, from 60 ft up to 1000 ft, and there is no particular spacing which seems to be most universally used.

Transverse contraction joints—Transverse contraction joints are generally used, although there are some states in which no contraction joints are specified. Slab lengths vary from 15 to 100 ft; 16 of the states used lengths of 15 to 20 ft in 1952, seven states used lengths of 70 to 100 ft; in those states using 30- to 40-ft lengths, there seems to be a trend away from the 30- and toward the 40-ft length.

As might be expected, generally when transverse joints are spaced at intervals of 20 ft or less, no reinforcement is required. However, there are several states not requiring reinforcement even when the joint spacing exceeds 20 ft, but these are in the minority.

Pavement width

There is a decided trend toward the use of wider pavements. A 24-ft pavement is now almost universally used with two 12-ft lanes, although

there are some states building 22-ft pavements. It is significant that not a single state has reported building pavement widths of 20 ft, whereas several years ago this width was still being constructed.

Thickness of pavement

So far as thickness is concerned, there is a definite trend away from the thickened-edge pavement. Thirty-three states now specify uniform cross section only. There is a definite trend toward thicker pavements; those constructing 9- and 10-in. pavements have changed from two states in 1948 to five states in 1952 and those states constructing 8-, 9-, and 10-in. pavements have changed from six states in 1948 to 18 states in 1952.

SELECTED REFERENCES

1. Westergaard, H. M., "Stress Concentrations in Plates Loaded Over Small Areas," *Proceedings*, ASCE, V. 68, Apr. 1942, pp. 509-534.

2. Westergaard, H. M., "New Formulas for Stresses in Concrete Pavements for Airfields," *Transactions*, ASCE, V. 113, 1948, pp. 425-444.

3. Pickett, G., and Ray, G. K., "Influence Charts for Concrete Pavements," *Proceedings*, ASCE, V. 76, Separate No. 12, May 1950.

4. *Design of Concrete Airport Pavement*, Portland Cement Association, Chicago, 1950.

5. *Concrete Pavement Design*, Portland Cement Association, Chicago, 1951.

6. Teller, L. W., and Sutherland, Earl C., "The Structural Design of Concrete Pavements," reprint by Public Roads Administration from *Public Roads;* V. 16: No. 8, 9, 10; V. 17: No. 7, 8; V. 23: No. 8.

7. Bradbury, Royall D., "Reinforced Concrete Pavements," Wire Reinforcement Institute, Washington, D. C., 1938.

8. Kelley, E. F., "Application of the Results of Research to the Structural Design of Concrete Pavement," *Public Roads*, V. 20, No. 5, July 1939, pp. 83-104.

9. Spangler, M. G., "Stresses in the Corner Region of Concrete Pavements," *Bulletin* No. 157, Iowa Engineering Experiment Station, Iowa State College, Ames, 1942.

10. Murphy, Glenn, "Stresses and Deflections in Loaded Rectangular Plates on Elastic Foundations," *Bulletin* No. 135, Iowa Engineering Experiment Station, Iowa State College, Ames, 1937.

For such discussion of this report as may develop please see June 1957 JOURNAL.

Title No. 53-25

Guide for Ultimate Strength Design of Reinforced Concrete*

By CHARLES S. WHITNEY and EDWARD COHEN†

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SYNOPSIS

This paper is intended to serve as a supplement to the ACI Building Code (ACI 318-56)[1] which permits the use of the ultimate strength method for the design of reinforced concrete members. It presents the method in its simplest form with working equations and charts to aid in their application. It aims to give the designing engineer all the information that he needs for the use of the ultimate strength method in accordance with the recommendations of the report of the ACI-ASCE joint committee on ultimate strength design.[2]

Regarding those problems not covered by the Code such as consideration of shear, bond, and deflections, recommendations are made based on the writers' practice.

INTRODUCTION

The term ultimate strength design means the design of reinforced concrete sections by the plastic theory to resist moments, thrusts, and shears which have been determined from elastic analysis of the structure under the assumed design loads multiplied by specified load factors. It is design for predetermined factors of safety against the actual ultimate strength of any section.

It should be noted that plastic theory is only a method of proportioning sections based on their actual strength as confirmed by tests. When combined with load factors, it provides a method of obtaining a uniform factor of safety. As a result, the designer is enabled to make a more rational structural design, and to avoid wasting construction material. While it saves design time and allows the designer some additional latitude in choice of sections, it does not relieve the designer of any of his previous responsibilities nor does it eliminate the need for experience and good judgment.

It is difficult to define the actual economies involved in design of structures by the ultimate strength method. However, it is safe to assume that because the members will be of more uniform strength, savings will be effected by eliminating excess material from sections where it is not actually needed. Any analysis should consider the savings in formwork, possible reduction of over-all story height where beam depths are reduced or haunches eliminated, and savings in foundations where the total weight of materials is reduced.

Among the structures where obvious major economies are possible are those in which important forces are developed by volumetric changes such as temperature, shrinkage, creep, etc. By reducing the size of members and thus reducing the rigidity of the structure, the stresses caused by the volumetric changes are minimized. Thus, not only are economies possible by using the minimum required material for a given set of moments and thrusts, but the design moments and thrusts are themselves reduced.

In the past, it had been assumed that a reliable factor of safety was obtained by designing for an allowable elastic stress which was generally taken as a yield stress divided by the so-called factor of safety. It is now realized that this is true only in isolated cases.

The design of beams under flexure only by the method based on working stresses and assumed straight-line variation of stress in the concrete can give

approximately correct results for lightly reinforced members, but it grossly underestimates the compressive flexural strength of the concrete. The use of the plastic theory therefore permits smaller, tougher beams, more heavily reinforced for tension with a reduction of compression reinforcement.

The factor of safety provided by the straight-line method may be much less than assumed. For example, if a member subjected to dead load compression and live load flexure is designed for working loads on the basis of allowable stresses, the factor of safety against an increase in live load may be far below the value assumed in selecting the allowable stress, because the dead load compression is fixed. The effect of increasing the live load moments without changing the dead load compression may be to increase the eccentricity disproportionately, thus resulting in unexpectedly high stresses. If the member were originally designed by the usual method, assuming proportional increase in thrust and moment due to overload, it might have little reserve for the tension stresses produced by the overload condition and failure could follow.

In prestressed concrete beams it is obvious that the initial stresses and the stresses under working load have no fixed relationship to overload capacity. After tension develops on one face, the member behaves essentially as a normal reinforced concrete beam and the ultimate strength can be computed without reference to the initial stress. In continuous members designed for more than one loading condition, it is generally recognized that there is usually some reserve capacity which can be developed after yielding at any one critical section.

The load factors recommended for ultimate strength design provide that the effect of the maximum internal forces acting on each section of a reinforced concrete structure will bear a uniform ratio to the ultimate strength of that section. It follows that the factors of safety used for the design of the individual sections will be the minimum factors of safety against collapse of the total structure, and will be the actual factors of safety against collapse only where the structure is a simple beam or column or a rigid frame with a fixed position of loads and free of temperature and other stresses from volumetric changes.

For a rigid frame structure designed for pattern loading or moving loads, the collapse load will be higher than any of the given design loads multiplied by the load factors. This is so because generally only one or two sections of the structure are stressed fully by any one position of the load. Therefore, as the load is increased these sections will yield without loss of strength, and other sections not fully stressed will be brought to their full capacity. Only when a complete mechanism action is developed, will failure take place. Many tests have indicated that reinforced concrete structures have more than sufficient ductility to form plastic hinges and allow such redistribution under heavy overload. This characteristic has recently been verified by severe tests of structures subjected to blast pressures from atomic bombs.[3]

An additional factor of safety against failure due to overload is available

in those structures where volumetric effects are present in the final design moments and thrusts. Because volumetric flexural stresses are self relieving, that is they tend to reduce and disappear as a result of yielding, they do not affect the overload capacity and actually add to the factor of safety against failure.

In addition to providing safety against failure from overload it is of prime importance that structures be designed to prevent objectionable cracking, fatigue failures, and excessive deflections under service conditions. Volumetric stresses may be very important under service load conditions as they may induce severe cracking or even failure in shear.

Load factors for ultimate strength design have been selected on the premise that the strength of the concrete in the structure will be not less than the assumed value of f_c'. To assure proper concrete strength, the concrete mix must be designed so that the average 28-day cylinder strength will be larger than f_c' by an amount, depending on the accuracy of control, sufficient to guarantee reliable strength in the structure.[4]

For normal control and unformity of field operations the average strength should be between 10 and 25 percent above the design strength. The 1956 ACI Code specifies for ultimate strength design that not more than one 28-day test in ten shall have an average strength less than the design strength, and the average of any three consecutive tests shall not be less than the design strength.

If at any time it appears that these conditions may not be met, an immediate investigation should be made of the cause and corrective measures taken.

The final phase of ultimate strength design involves the proportioning of sections to resist the computed moments, thrusts, and shears. For practical use in design offices the ultimate strength formulas of the report[2] of the ACI-ASCE joint committee on ultimate strength and the ACI Building Code (ACI 318-56),[1] applicable to an assumed equivalent rectangular stress block, have been collected and presented in their simplest forms in this paper.

DESIGN ASSUMPTIONS AND NOMENCLATURE

The formulas in this paper are based on the following design assumptions applying to the equivalent rectangular stress block method, and are in conformance with the general assumptions stated in the report of the joint committee on ultimate strength:

(a) Tensile strength in concrete is neglected in the design of sections subject to bending.

(b) At ultimate load, stresses and strains are not proportional and the distribution of compression stresses in a section subject to bending is nonlinear. The compressive stress distribution used for design is assumed to be rectangular.

(c) The uniform compressive stress in the equivalent rectangular stress block is equal to $0.85 f_c'$.

(d) The total force and the location of the centroid in the rectangular stress block are the same as for the actual nonlinear stress distribution.

The following nomenclature is uniform with that of the joint committee report on ultimate strength:

A_g = gross cross-sectional area
A_s = tensile steel area
A_s' = compressive steel area
A_{st} = total steel area = $A_s + A_s'$
a = depth of rectangular stress block
b = width of rectangular beam or column or total width of T-beam flange
b' = width of stem of T-beam
C = total compression in beam or in column carrying moment
D = outside diameter of circular column
d = distance from extreme compressive fiber to centroid of tension force in tensile reinforcement; also, in a column with a circular core, the diameter of the circle passing through the centers of reinforcing bars
d' = distance from extreme compressive fiber to centroid of compressive reinforcement
e = eccentricity of axial load measured from centroid of tensile steel
e' = eccentricity of axial load measured from the center of the section for sections with symmetrical reinforcement; also, the eccentricity of the axial load from the plastic centroid of rectangular sections with unsymmetrical reinforcement
f_c' = 28-day concrete cylinder strength
f_y = yield point stress in steel, but not greater than 60,000 psi for design purposes
f_y' = $f_y - 0.85\, f_c'$ = effective yield point stress in compressive reinforcement after making a deduction for the displaced concrete
l = unsupported length of an axially or eccentrically loaded compression member
M_u = ultimate bending moment
m = $f_y/0.85\, f_c'$
m' = $m - 1$
P_o = ultimate direct load capacity for a concentrically loaded short column
P_u = ultimate direct load capacity for an eccentrically loaded short column
P_u' = ultimate direct load capacity for an eccentrically loaded long column
p = A_s/bd
p' = A_s'/bd
p_t = $(A_s + A_s')/bt = (A_s + A_s')/A_g$
T = total tension
t = total thickness of slab or column

LOAD FACTORS

The following terms are used in the load factor equations:

U = ultimate strength of section—the maximum combination of thrust and moment the member can sustain prior to failure
B = effect of basic load consisting of dead load (plus volume change due to plastic and elastic actions, shrinkage and temperature for continuous frames, arches and similar structures)
L = effect of live load plus impact
W = effect of wind load
E = effect of earthquake forces
K = load factor equal to 2.0 for columns and members subjected to combined bending and axial load, and equal to 1.8 for beams and girders subjected to bending where the axial load is negligible.

The effect of a load is considered to be the combination of thrust (or tension) and bending moment produced by the load.

Members should be proportioned so, that: (1) they will be capable of carrying without failure the critical load combination given below, thereby insuring an ample factor of safety against an increase in live load beyond that assumed in design; (2) the strains under working loads will not be so large

as to cause excessive cracking. These criteria are satisfied if the sections are designed for the loading combinations given by the following formulas:

For those structures for which, due to location or proportions, the effects of wind and earthquake loading can properly be neglected:

$$U = 1.2B + 2.4L \quad \text{(I)}$$

$$U = K(B + L) \quad \text{(II)}$$

For those structures in which wind loading should be considered:

$$U = 1.2B + 2.4L + 0.6W \quad \text{(Ia)}$$

$$U = 1.2B + 0.6L + 2.4W \quad \text{(Ib)}$$

$$U = K\left(B + L + \frac{W}{2}\right) \quad \text{(IIa)}$$

$$U = K\left(B + \frac{L}{2} + W\right) \quad \text{(IIb)}$$

For those structures in which earthquake loading should be considered, substitute E for W in the preceding equations. In case there is doubt as to the importance of wind or earthquake loading it can be tested by making a trial calculation using these equations.

RECTANGULAR BEAMS OR SLABS WITH TENSILE REINFORCEMENT ONLY

To determine the ultimate strength of a beam in flexure, the actual nonlinear stress block is replaced for simplicity, with a rectangular stress block of equal total force with an average stress intensity equal to 0.85 f_c'. The use of the rectangle is merely a mathematical device to approximate the effect of the true distribution.[5] It would be possible to use any curved shape that would give the same area and center of gravity, but the rectangular area (Fig. 1) gives the simplest mathematical solution. The following formulas are all based on this method and give good agreement with tests. The assumption of a concrete stress equal to 0.85 f_c' corresponds to the actual concrete strength for axially loaded columns and permits a consistent treat-

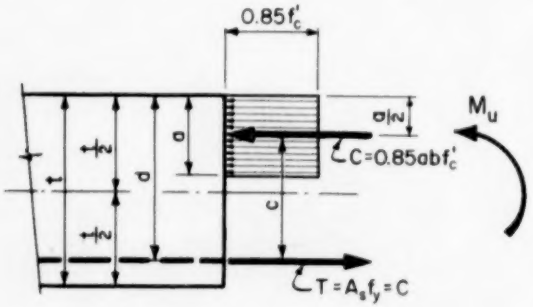

Fig. 1

ment through the full range from pure flexure (eccentricity, $e = \infty$) to pure axial load ($e = 0$).

It should be noted that the depth a is not the same as the depth to the neutral axis, $k_u\, d$. The value of $k_u\, d$ is determined by the strains at the top and bottom of the beam whereas a is determined by the strengths of the materials and the area of reinforcement.

For practical purposes, the action of the beam may now be described as follows:

If the beam is under-reinforced so that the primary failure will occur in the tensile steel, the concrete will crack as the steel stretches, and the equivalent depth, a, of the beam in compression will decrease until the average effective concrete stress reaches $0.85\, f_c'$ and the lever arm of the resisting couple reaches a maximum. If the applied moment is increased, the beam will then fail progressively by crushing of the concrete.

Therefore:

$$a = \frac{A_s f_y}{0.85 b f_c'} = \frac{A_s m}{b} \qquad (1)$$

or

$$\frac{a}{d} = p\, m \qquad (2)$$

The lever arm of the steel reinforcement is then

$$c = d - \frac{a}{2} = d - \frac{A_s m}{2b} \qquad (3)$$

or

$$\frac{c}{d} = 1 - \frac{p\, m}{2} \qquad (4)$$

The ultimate resisting moment of the beam as controlled by yield of the tensile steel can be written:

$$M_u = c A_s f_y = A_s f_y \left[d - \frac{A_s m}{2b} \right] \qquad (5)$$

$$\frac{M_u}{b\, d^2} = p f_y \left[1 - \frac{p\, m}{2} \right] \qquad (6)$$

For under-reinforced beams the following relationships can be established also:

$$\frac{a}{d} = 1 - \sqrt{1 - \frac{2.35\, M_u}{f_c'\, b\, d^2}} \qquad (7)$$

$$\frac{c}{d} = \frac{1}{2} \left[1 + \sqrt{1 - \frac{2.35\, M_u}{f_c'\, b\, d^2}} \right] \qquad (8)$$

$$p_{req'd.} = \frac{1}{m} - \sqrt{\frac{1}{m^2} - \frac{2 M_u}{f_y m\, b\, d^2}} \qquad (9)$$

For special cases where the tensile reinforcement is distributed across the depth of the section the steel in the outer quarter of the depth near the tension face may be considered fully effective, the depth being measured from the extreme compression fiber to the extreme tension bar. Because of the reduction in strain as the neutral axis is approached, the stress in those bars outside this region may be assumed to vary linearly from zero at the neutral axis to f_y at the quarter point near the tension face. The neutral axis may be taken at a distance of $1.2a$ from the compression face of the section. The depth, d, to be used for calculating a and M_u is measured from the extreme compressive fiber to the resultant of the forces in the tensile reinforcement.

It has been established experimentally that the average maximum value of M_u/bd^2 for the case of balanced reinforcement is

$$\frac{M_u}{bd^2} = \frac{f_c'}{3} \quad \ldots\ldots (10)$$

The values of a/d and p corresponding to the above are given by

$$a/d = 0.537 \quad \ldots\ldots (11)$$

and

$$p = 0.456 \frac{f_c'}{f_y} \quad \ldots\ldots (12)$$

Further increase in the percentage of steel will result in primary compression failure in the concrete without yielding of the steel and without substantial increase in M_u/bd^2. If the steel percentage is less than that given by Eq. (12), the moment capacity will be governed by yielding of the steel at a value of M_u/bd^2 as given by Eq. (6). The area of steel required for balanced reinforcement is so large that it is usually neither practical nor economical to develop the full flexural capacity of the concrete section.

To ensure against sudden compression failures, the ACI Code requires that in flexural members the ratio of tension reinforcement shall not be greater than

$$p = 0.4 \frac{f_c'}{f_y} \quad \ldots\ldots (13)$$

This results in a maximum compression block to depth ratio of

$$a/d = 0.47 \quad \ldots\ldots (14)$$

and a maximum allowable design moment without compression steel of

$$\frac{M_u}{bd^2} = 0.306 f_c' \quad \ldots\ldots (15)$$

For the same reason it is required that the coefficient 0.40 in Eq. (13)

shall be reduced by $6.25[(f_c' - 5000)/1000]$ percent for concrete strength in excess of 5000 psi. Corresponding reductions are therefore required for a/d from Eq. (14) and M_u/bd_3 from Eq. (15). These reductions also apply to all following equations where these terms are used.

Chart 1 is a universal design chart giving the ultimate bending moments for rectangular beams for all values of p, f_c', and f_y.

RECTANGULAR BEAMS OR SLABS WITH COMPRESSION REINFORCEMENT

If enough tensile steel is supplied to develop the full compression strength, the maximum moment permitted on a beam reinforced for compression is

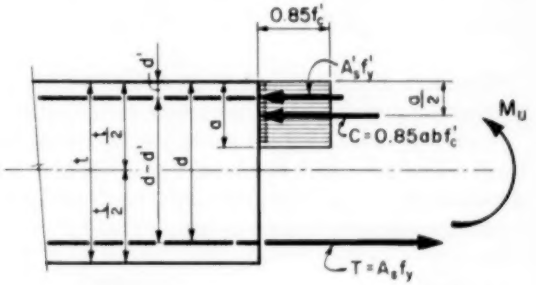

Fig. 2

obtained by adding the moment of the compression steel to that of the concrete compression stress block (Fig. 2).

$$\frac{M_u}{bd^2} = 0.306 f_c' + p' f_y' \left(1 - \frac{d'}{d}\right) \quad \text{.....(16)}$$

The required ratio of tension reinforcement may be computed as

$$p = 0.4 \frac{f_c'}{f_y} + p' \frac{f_y'}{f_y} \quad \text{.....(17)}$$

The maximum ratio of tensile reinforcement shall be so limited that $(p - p')$ does not exceed the value given by Eq. (13).

Since this will usually require more tensile steel than can be readily placed in the beam, compression steel will not normally be used. However, it may be present for other reasons, such as providing continuity, providing for reversing moments, or reducing creep and deflections. It may also be necessary to compute the ultimate moment capacity governed by compression for the design of sections subject to combined bending and direct stress.

When the compressive reinforcement is placed in more than one layer or is distributed along all the faces, the bars between the compression face and $d/4$ from the compression face may be considered fully effective when the moment capacity is governed by the compression strength. The stress in those bars located between $d/4$ and $d/2$ from the compression face may be

assumed to vary from zero at the center to f_y at the quarter point. The effective depth, d, is measured from the extreme compression fiber to the resultant of the tension forces in the tensile reinforcement. When the tension reinforcement also is placed in several layers or is distributed along all the faces, the resultant of the tension forces may be determined as described previously (p. 460 ff.).

When the strength of a beam is limited by the tension steel, it may be designed as if the compression steel were not present. However, if the compression steel is nearer the compression face than the center of the concrete compression stress block, the tensile steel required can be reduced by taking advantage of the longer lever arm by using the following formula.

$$\frac{M_u}{bd^2} = (p - p')\left(1 - \frac{(p - p')m}{2}\right)f_y + p' f_y \left(1 - \frac{d'}{d}\right) \quad \ldots\ldots\ldots\ldots (18)$$

Although Eq. (18) makes no deduction for the area of concrete displaced by the compression steel, the error involved is not significant.

This case can be solved also by subtracting the moment of the compressive steel from the total moment. The tension steel requirement will be that obtained from Eq. (16) or Chart 1 for this net moment plus an amount equal to the compressive reinforcement.

T-BEAMS

Since it will not usually be possible to place enough tensile steel in the stem of a T-beam to develop the full compressive strength of the flange, the effect of a flange is principally to increase the lever arm of the internal resisting couple and reduce the amount of tension steel below that required for the beam without a flange (Fig. 3). The report of the ultimate strength

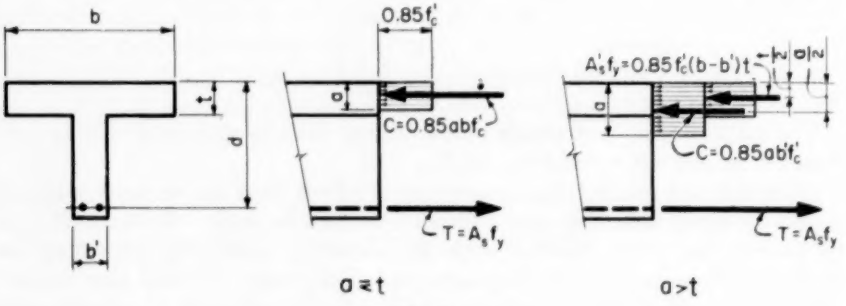

Fig. 3

committee[2] limits the effective width of flange on either side of the stem to a maximum of six times the slab thickness.

If the depth of the concrete compression stress block is equal to or less than the slab thickness, that is $a = pmd \leqq t$, the beam may be designed as a rectangular beam using Chart 1.

If the equivalent compression stress block extends into the stem below the bottom of the slab, the T-beam can be designed as a rectangular beam with compression reinforcement by considering the flanges as compression reinforcement, acting at the middepth of the slab with an equivalent compression steel area equal to

$$A_s' = 0.85 f_c' (b - b') \frac{t}{f_y} = \frac{(b - b') t}{m} \quad \text{.....(19)}$$

CONCENTRICALLY LOADED SHORT COLUMNS

Controlled laboratory tests on concentrically loaded short columns have indicated that the maximum load capacity is given by

$$P_o = 0.85 f_c' (A_g - A_{st}) + A_{st} f_y = 0.85 f_c' A_g + A_{st} f_y' \quad \text{.....(20)}$$

However, in actual structures all members subject to axial load shall be designed for a minimum accidental eccentricity which may occur due to end conditions, inaccuracy of manufacture, or variation in materials even if the load is theoretically concentric. For spirally reinforced columns, the minimum eccentricity measured from the centroidal axis shall be taken as 0.05 times the total diameter of a round column section or 0.05 times the total thickness of a square section. For rectangular or square tied columns the minimum eccentricity along either of the main rectangular axes shall be taken separately as 0.10 times the least dimension of the section in the direction of the eccentricity.

Based on this requirement the maximum ultimate axial load capacity for symmetrically reinforced column sections is given below. These equations may be obtained from those given for combined bending and axial compressive load (p. 466 ff.).

Rectangular tied columns

$$P_u = \frac{2 A_s' f_y'}{\frac{0.2t}{(d - d')} + 1} + \frac{bt f_c'}{\frac{0.3t^2}{d^2} + 1.18} \quad \text{.....(21)}$$

Square column, round core, spiral reinforcement

$$P_u = \frac{A_{st} f_y'}{\frac{0.15 D}{d} + 1} + \frac{A_g f_c'}{\frac{0.6 t^2}{(t + 0.67d)^2} + 1.18} \quad \text{.....(22)}$$

Circular column, round core, spiral reinforcement

$$P_u = \frac{A_{st} f_y'}{\frac{0.15D}{d} + 1} + \frac{A_g f_c'}{\frac{0.48 D^2}{(0.8D + 0.67d)^2} + 1.18} \quad \text{.....(23)}$$

The ultimate axial load capacity for the above three cases may be obtained also from Charts 2 and 3.

COMBINED BENDING AND COMPRESSIVE AXIAL LOAD

The ultimate strength of members under combined bending and axial load may be determined from the formulas of this section. When the ultimate thrust-moment strength is controlled by yield of the tension reinforcement (tension failure), the formulas are derived directly from the equations of equilibrium and the principles developed in sections on rectangular beams or slabs (see p. 460 ff. and p. 463 ff.).

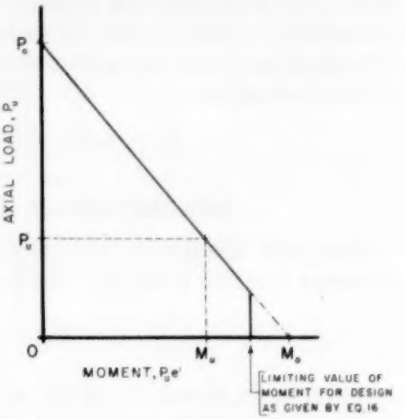

Fig. 4—Linear relationship between axial load and moment for compression failure

When the strength is controlled by crushing of the concrete and yield of the compression reinforcement (compression failure), the formulas are based on a straight line relationship (Fig. 4) between the cases of pure axial load, ($M_u = P_u e' = 0$, $P_u = P_o$), and pure bending, ($M_u = M_o$, $P_u = 0$) as given by

$$\frac{M_u}{M_o} = \frac{P_o - P_u}{P_o} \quad \text{.....(24)}$$

where M_u is the total moment about the plastic centroid of the section. This relationship is in good agreement with test results.

The plastic centroid is defined as the location of the resultant of the forces in the concrete and the reinforcing steel when they are stressed uniformly to 0.85 f_c' and f_y respectively.

M_o is the moment capacity without thrust as controlled by compression assuming enough tensile steel to develop it in full. For a rectangular section this moment is given as

$$M_o = 0.333\, b\, d^2 f_c' + A_s' f_y' (d - d') \quad \text{.....(24a)}$$

However, the maximum moment available for design purposes with low values of axial load is given by Eq. (16). For concrete strengths in excess of 5000 psi, the coefficient 0.333 is to be reduced in the same manner as the coefficient 0.306 in Eq. (15).

M_o may be defined in a similar manner for round sections and square sections with round cores.

Rectangular section (Fig. 5)

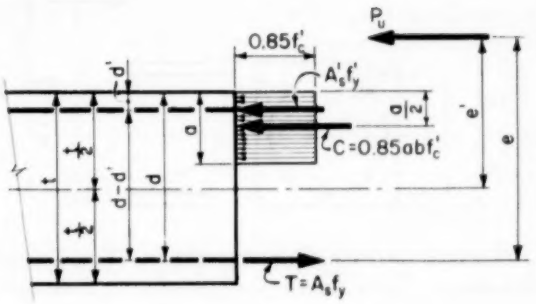

Fig. 5

Strength controlled by tension—When the ultimate load is controlled by tension in the steel

$$P_u = 0.85f_c'bd\left\{p'm' - pm + 1 - \frac{e}{d} + \sqrt{\left(1 - \frac{e}{d}\right)^2 + 2\left[\frac{e}{d}(pm - p'm') + p'm'\left(1 - \frac{d'}{d}\right)\right]}\right\} \quad (25)$$

For symmetrical reinforcement this reduces to

$$P_u = 0.85\,f_c'bd\left\{1 - p - \frac{e}{d} + \sqrt{\left(1 - \frac{e}{d}\right)^2 + 2p\left[m'\left(1 - \frac{d'}{d}\right) + \frac{e}{d}\right]}\right\} \quad (26)$$

Chart 4 and Charts 5-8 give ultimate strengths based on Eq. (26). With no compression reinforcement this further reduces to

$$P_u = 0.85\,f_c'\,bd\left\{1 - pm - \frac{e}{d} + \sqrt{\left(1 - \frac{e}{d}\right)^2 + 2\frac{epm}{d}}\right\} \quad (27)$$

Strength controlled by compression—For symmetrically reinforced rectangular sections the following design formula is obtained where the strength is limited by compression

$$P_u = \frac{2A_c'f_y'}{\frac{2e'}{(d - d')} + 1} + \frac{b\,t\,f_c'}{\frac{3te'}{d^2} + 1.18} \quad (28)$$

The solution of this formula is given also in Charts 4-7, except that no deduction is made for the concrete displaced by the compression reinforcement. Where compression controls, allowance may be made by substituting p_tm' for p_tm in using the charts.

Square section, round core (Fig. 6)

Strength controlled by tension—For ultimate load producing tension failure:

$$P_u = 0.85f_c'\,t^2\left\{\sqrt{\left(\frac{e'}{t} - 0.5\right)^2 + 0.67\,\frac{d}{t}\,p_t\,m} - \left(\frac{e'}{t} - 0.5\right)\right\} \quad (29)$$

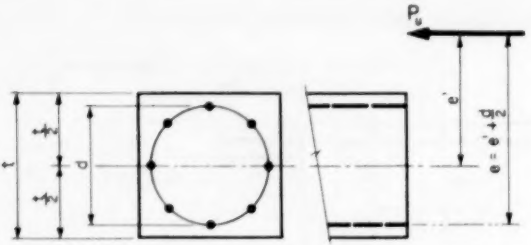

Fig. 6

Strength controlled by compression—For a square section with spiral reinforcement, the ultimate load producing a compression failure is:

$$P_u = \frac{A_{st} f_y'}{\frac{3e'}{d} + 1} + \frac{A_g f_c'}{\frac{12.0\, t e'}{(t + 0.67d)^2} + 1.18} \quad \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

The solution of Eq. (29) and (30) is given on Charts 8-11. Where compression controls, $p_t m'$ may be substituted for $p_t m$ to make a deduction for concrete displaced by compression reinforcement.

Circular section, round core (Fig. 7)

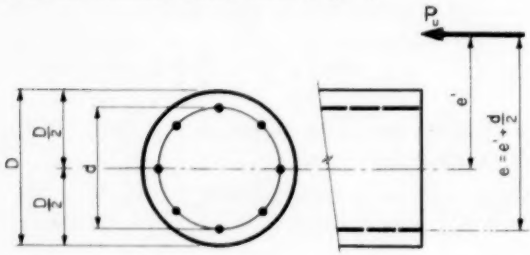

Fig. 7

Strength controlled by tension—For a round column with a round core, the ultimate load producing a tension failure is

$$P_u = 0.85 f_c' D^2 \left\{ \sqrt{\left(\frac{0.85e'}{D} - 0.38 \right)^2 + \frac{dp_t m}{2.5D}} - \left(\frac{0.85e'}{D} - 0.38 \right) \right\} \quad \ldots\ldots (31)$$

Strength controlled by compression—The ultimate load producing a compression failure is given by

$$P_u = \frac{A_{st} f_y'}{\frac{3e'}{d} + 1} + \frac{A_g f_c'}{\frac{9.6\, De'}{(0.8D + 0.67d)^2} + 1.18} \quad \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

Charts 12-15 give the ultimate strength of round columns with round cores. Where compression controls Eq. (32), $p_t m'$ may be substituted for $p_t m$ to make a deduction for concrete displaced by compression reinforcement.

Axial load with bending about both axes

The case of combined axial load and bending about both axes can be solved by using a uniform stress of $0.85f_c'$ over the entire compression block (Fig. 8).

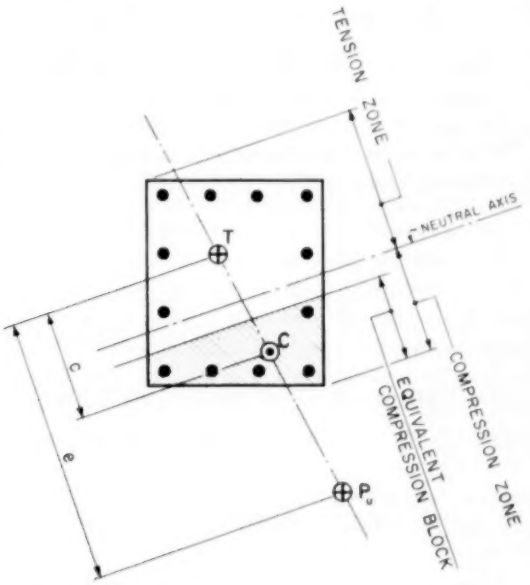

Fig. 8—Rectangular section with axial load plus bending about both axes

The solution must satisfy the following conditions:

1. The resultant of the tensile forces in the steel, T; the resultant of the compression forces in the compression steel and concrete, C; and the applied load, P_u, must all lie in the same plane.
2. The forces in the vertical direction must be in equilibrium, $P_u = C - T$.
3. The moments in the plane of the resultant forces must be in equilibrium, $P_u e = Cc$ or $Tc = P_u (e - c)$.

For non-rectangular sections and to check whether compression governs it is desirable to check the solution by using a maximum allowable strain, $\epsilon = 0.003$, for the concrete in compression and assuming a linear strain distribution along a line perpendicular to the neutral axis.

The problem of unsymmetrical bending without axial load may be solved in a similar manner by making the planes of the existing moment and the applied moment coincident.

COMBINED BENDING AND TENSILE AXIAL LOAD

The ultimate strength of members under combined bending and axial tension may be determined from the equations given below. When the resultant falls within the depth of the section, the entire section will be under tension; these formulas do not hold, and tension reinforcement must be provided on both faces.

Rectangular section (Fig. 9)

Strength controlled by tension—With no compression reinforcement the ultimate load may be expressed as

$$P_u = 0.85 f_c' bd \left\{ pm - 1 - \frac{e}{d} + \sqrt{\left(1 + \frac{e}{d}\right)^2 - 2\left(\frac{e}{d}\right) pm'} \right\} \quad \ldots\ldots (33)$$

With compression reinforcement and $A_s f_y - A_s' f_y' > P_u$ the ultimate load may be expressed as

$$P_u = 0.85 f_c' bd \left\{ pm - p'm' - 1 - \frac{e}{d} + \sqrt{\left(1 + \frac{e}{d}\right)^2 - 2\left[\frac{e}{d}(pm - p'm') - p'm'\left(1 - \frac{d'}{d}\right)\right]} \right\} \quad \ldots\ldots (34)$$

The depth of the compression block is given by

$$a = \frac{A_s f_y - A_s' f_y' - P_u}{0.85 f_c' b} \quad \ldots\ldots (35)$$

If $a \leqq d'$ compressive reinforcement is not required and Eq. (33) should be used.

When $A_s f_y - A_s' f_y' \leqq P_u$ the ultimate load is given by

$$P_u = \frac{A_s f_y}{\frac{e}{d - d'} + 1} \quad \ldots\ldots (36)$$

Strength controlled by compression—When the ultimate load capacity is controlled by compression

$$P_u = \frac{1}{e}\left[A_s' f_y' (d - d') + 0.306 f_c' bd^2 \right] \quad \ldots\ldots (37)$$

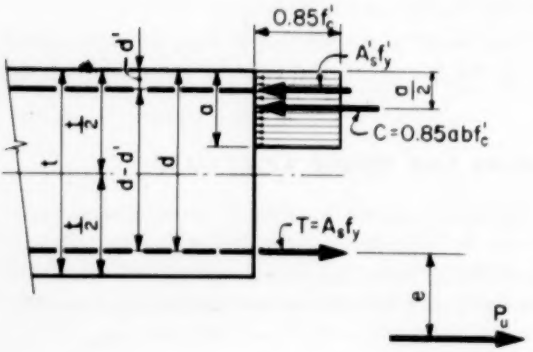

Fig. 9

SLENDER COMPRESSION MEMBERS

When the unsupported length, l, of an axially or eccentrically loaded compression member is greater than 15 times its least lateral dimension, the maximum axial load can be determined by one of the following methods:

1. $$P_u' = P_u\,(1.6 - 0.04l/t) \quad \ldots\ldots\ldots\

tions regarding the ultimate strength of reinforced concrete members in these two items are proposed at this time." In the absence of code provisions the writers present the following recommendation based on their personal appraisal of the latest investigations.

For consistency with the flexural calculations, design for shear and bond should be based on the same load factors used for the computation of thrust and moment. The allowable stresses for shear and bond shall then be multiplied by the same value of K* used for the flexural design.

Shear (diagonal tension)

In using the ultimate strength method of design, the primary objective is to maintain the integrity of the member up to the breaking load. This means that the full flexural strength of the critical sections as controlled by the principal reinforcing should be developed before failure occurs in bond or shear. This permits the formation of plastic hinges and the readjustment of moments in continuous beams and frames before reaching ultimate load. It would obviously be unsatisfactory design if sufficient web reinforcement is not provided to permit full utilization of the longitudinal steel.

Since the ultimate strength method involves the use of higher stresses in longitudinal reinforcing than permitted by the standard "straight line" method, careful consideration must be given to details of design to obtain maximum benefit. One of the inherent advantages of the ultimate strength method is that it permits the use of tougher and more flexible members resulting in a reduction of volume change forces. However, observations in both field and laboratory have proven that volume changes in the concrete, due principally to shrinkage and temperature changes, can have serious and unpredicted effects on the diagonal tension strength by producing initial tensions and cracking in the unloaded members. It is important that careful consideration be given to the effect of longitudinal tension due to restraint of columns in continuous frames and that the longitudinal and shear reinforcement be made adequate to resist the resultant forces.

If the longitudinal tension can be estimated, its component should be considered together with the shear stress for the design of web reinforcement.

For the design of web reinforcement, the writers suggest that Chapter 8 of the ACI Building Code (ACI 318-56), be used with the following modification.

It is recommended that the following be added to Section 801 (d):

"Web reinforcement shall be provided from the support to a point beyond the extreme position of the point of inflection a distance equal to either 1/16 of the clear span or the depth of the member, whichever is greater even though the shearing unit stress does not exceed v_c. Web reinforcement shall be provided at every section in which negative reinforcement is required. Where required by this paragraph, the amount of

*See Load Factors, p. 459.

web reinforcement at each section shall be the maximum required by any one of the following:

1. Sufficient to carry 2/3 of the total shear where the unit stress exceeds v_c.
2. Sufficient to carry 2/3 of the total shear existing at the point of inflection, that is, the ratio of web reinforcement required at the point of inflection will be maintained back to the support.
3. Not less than 0.15 percent of the area computed as the product of the width of member at middepth and the horizontal spacing of the web reinforcement."

This addition is not intended to apply to small T-beams forming part of a joist floor construction. The use of bent-up bars for diagonal reinforcement is desirable and should be used where practical.

Bond

Deformed bars (ASTM A 305) are very efficient in developing bond and reducing the width of tension cracks as long as longitudinal splitting of the concrete along the bar is prevented. It is therefore important that lateral ties be provided to prevent such splitting especially where bond stresses are high.

Since it is recognized that compressive steel may work at yield point stresses, it is also important that lateral ties be provided to prevent buckling of compression steel.

It is therefore recommended that lateral reinforcement be provided in the form of stirrups or ties, wherever reinforcing bars are present on both the tension and compression surfaces of a member, where there is compressive steel, or where substantial bond stress may exist. These ties or stirrups should fully enclose the longitudinal reinforcing unless adequate restraint is provided by a concrete slab, as at the top of a T-beam.

The writers further recommend that the provisions of the ACI Building Code (ACI 318-56), Chapter 9, be followed with the following addition to Section 902 (b):

> "Not less than one-half of the area of the positive reinforcement shall extend toward the support far enough to lap with not less than one-third of the total area of the negative reinforcement on the opposite face for a distance not less than the depth of the beam."

Even though the positive and negative steel are on the opposite sides of the member, this lap will help to resist longitudinal tension due to volume changes. The web reinforcement recommended above will tie the positive and negative steel together and resist stripping of the main reinforcement if transverse cracks develop.

DEFLECTIONS

Undesirable deflections which have occurred in some structures designed by the straight-line method are not due to the method of design nor, in some cases, to stress conditions caused by dead or live load. The ultimate strength method of design will encourage the use of more slender members with steel working at higher stresses, thus increasing flexibility, and making careful

consideration of deflections more important than ever before. Because of the variety of conditions which affect the problem, such as end restraint, continuity, type of construction, loading, and occupancy, it is not possible to draft simple rules for proportioning concrete members to prevent excessive deflections.

Deflections due to load may be substantially increased by unsymmetrical shrinkage of the concrete. The strain in concrete due to shrinkage may exceed that due to a compression stress of 1000 psi. Since this strain may correspond to a stress of approximately 10,000 psi in the steel, an appreciable curvature may be caused in a beam which is unsymmetrically reinforced. Differential shrinkage may also be caused shortly after the concrete is placed by the different rates of loss of the heat of hydration in thin and thick sections. Curvature may be magnified by the more rapid shrinkage in a thin slab as compared with the more massive beams. Shrinkage deflections usually stabilize after the first year or two of the life of a structure.

Shrinkage deflections can be reduced by the use of:

1. Types of structure which do not have excessively shallow members and thin sections
2. Symmetrical reinforcement
3. Continuous restrained members
4. Concrete mix without excessive amounts of either water or very fine material in aggregate
5. Proper curing of concrete

Reasonably close estimates of deflections due to loading under working conditions can be made by use of the elastic theory.[6] A quick estimate on the high side can be made by using the gross moment of inertia of the concrete section, neglecting cracking and steel area, using a value of E_c of 1,000,000 psi or 750,000 psi for permanent load to allow for plastic flow as compared to 4,000,000 psi or 3,000,000 psi for temporary live loads. The use of compression reinforcement has been found to be very effective in reducing deflections under permanent load.[7,8] Where compression steel is present the deflection may be estimated by using the uncracked transformed section including the effect of the compression steel with $E_s = 30{,}000{,}000$.

The amount of deflection which can be safely allowed in any particular case requires consideration of the special conditions which control in that case. What might be perfectly satisfactory in the case of a long open span might be very objectionable under partitions or over windows. Usually members of minimum depth with the maximum amounts of reinforcement are not economical and should be used only where additional rigidity is not required. However, it should not be inferred that flexibility is necessarily objectionable if kept within proper limits.

ACKNOWLEDGMENTS

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Laing for their advice during the preparation of the material and to the staff of the Special Structures Section of Ammann & Whitney for their assistance in preparing the design charts and checking the manuscript.

REFERENCES

1. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-56)," ACI JOURNAL, May 1956, *Proc.* V. 52, pp. 913-986.

2. "Report of ASCE-ACI Joint Committee on Ultimate Strength Design," *Proceedings*, ASCE, V. 81, Paper No. 809, Oct. 1955; ACI JOURNAL, Jan. 1956, *Proc.* V. 52, pp. 505-524.

3. Whitney, C. S., Anderson, B. G., and Cohen, E., "Design of Blast Resistant Construction for Atomic Explosions," ACI JOURNAL, Mar. 1955, *Proc.* V. 51, pp. 589-684.

4. ACI Committee 214, "Evaluation of Compression Test Results of Field Concrete," ACI JOURNAL, Nov. 1955, *Proc.* V. 52, pp. 241-258.

5. Whitney, Charles S., "Plastic Theory of Reinforced Concrete Design," *Transactions*, ASCE, V. 107, 1942, p. 251: *Proceedings*, Dec. 1940.

6. "Deflection of Reinforced Concrete Members," *Publication* No. ST70, Portland Cement Association, Chicago, 1947.

7. Washa, G. W., and Fluck, P. G., "Effect of Compressive Reinforcement on the Plastic Flow of Reinforced Concrete Beams," ACI JOURNAL, Oct. 1952, *Proc.* V. 49, pp. 89-108.

8. Washa, G. W., and Fluck, P. G., "Plastic Flow (Creep) of Reinforced Concrete Continuous Beams," ACI JOURNAL, Jan. 1956, *Proc.* V. 52, pp. 549-562.

For such discussion of this paper as may develop please see June 1957 JOURNAL.

Enlargements of the following design charts (approximately 12x12 in.) are available in sets only at $2.00 per set of 15 charts. Order from ACI headquarters.

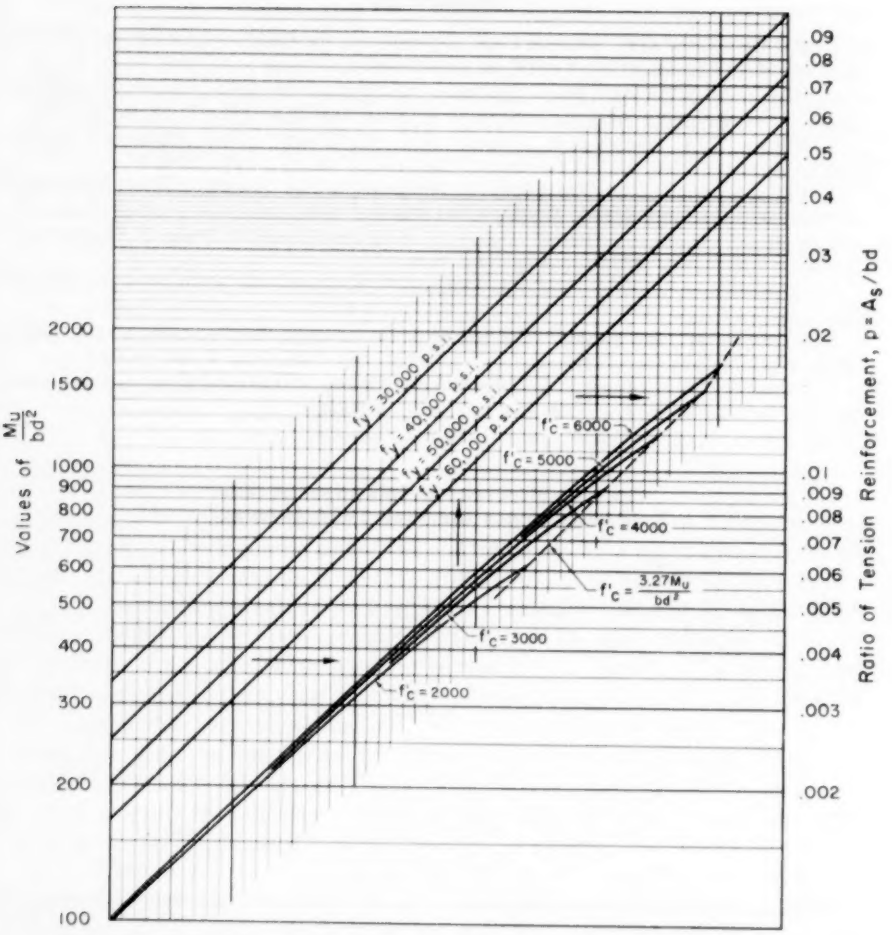

Chart 1—Moment capacity of rectangular sections without compression reinforcement

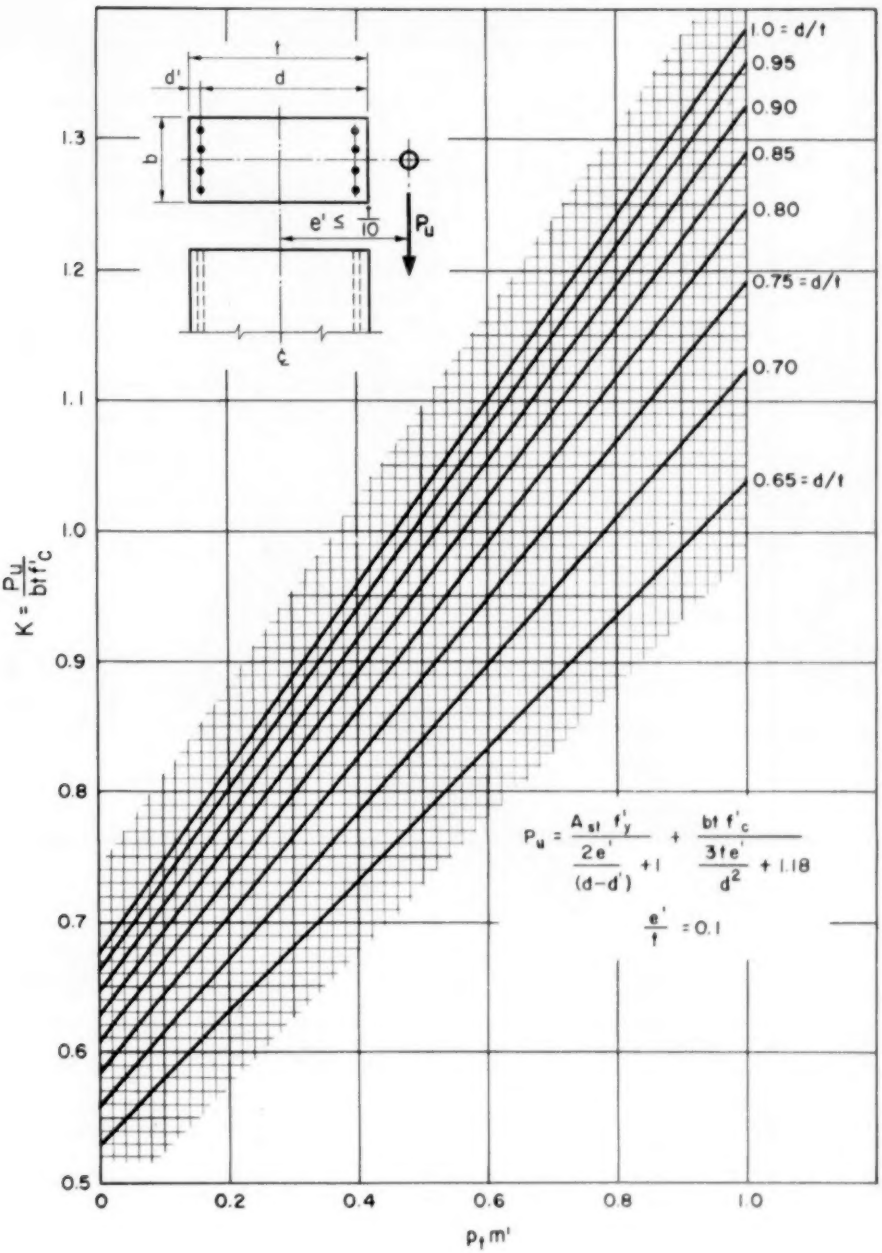

Chart 2—Direct load capacity of rectangular sections with symmetrical reinforcement, for eccentricities less than or equal to $t/10$

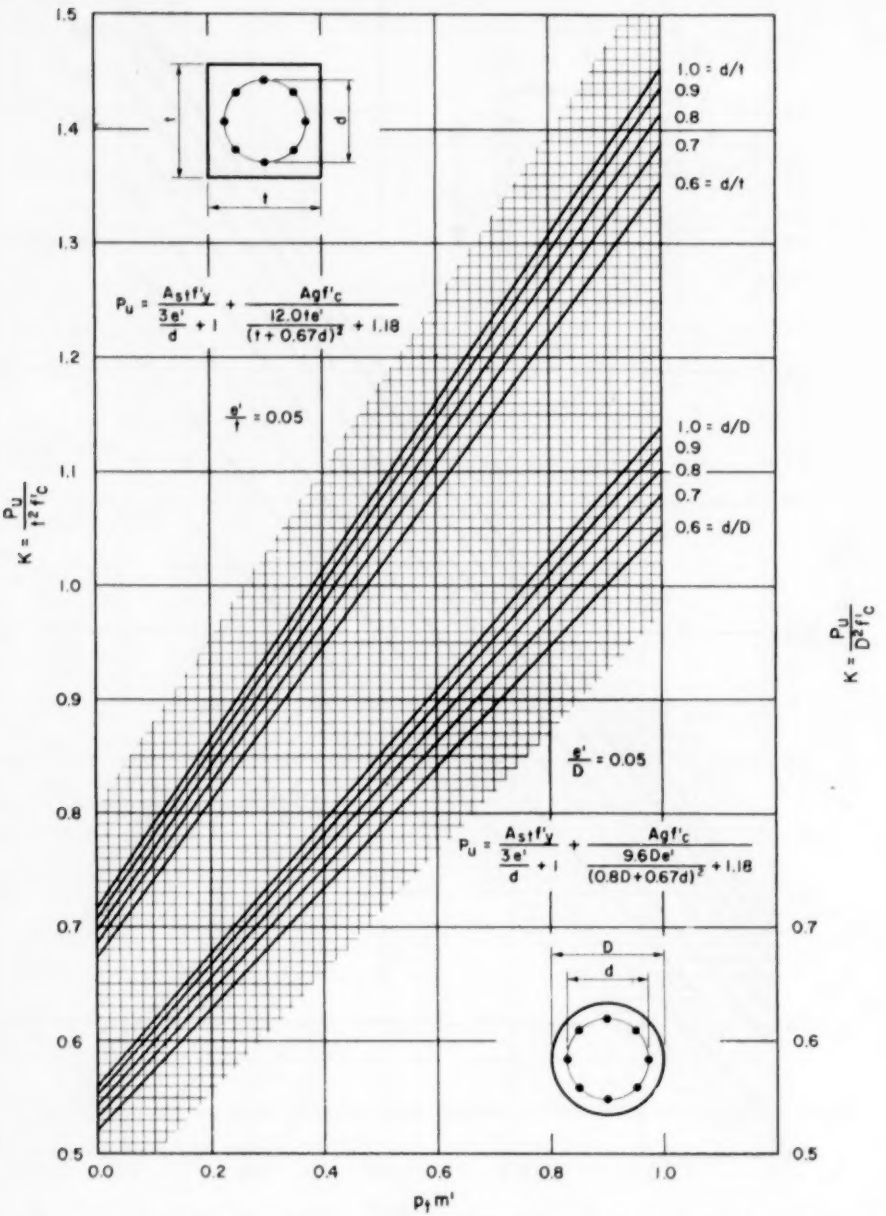

Chart 3—Direct load capacity of square sections with spiral reinforcement and circular sections with spiral reinforcement for eccentricities less than or equal to $t/10$

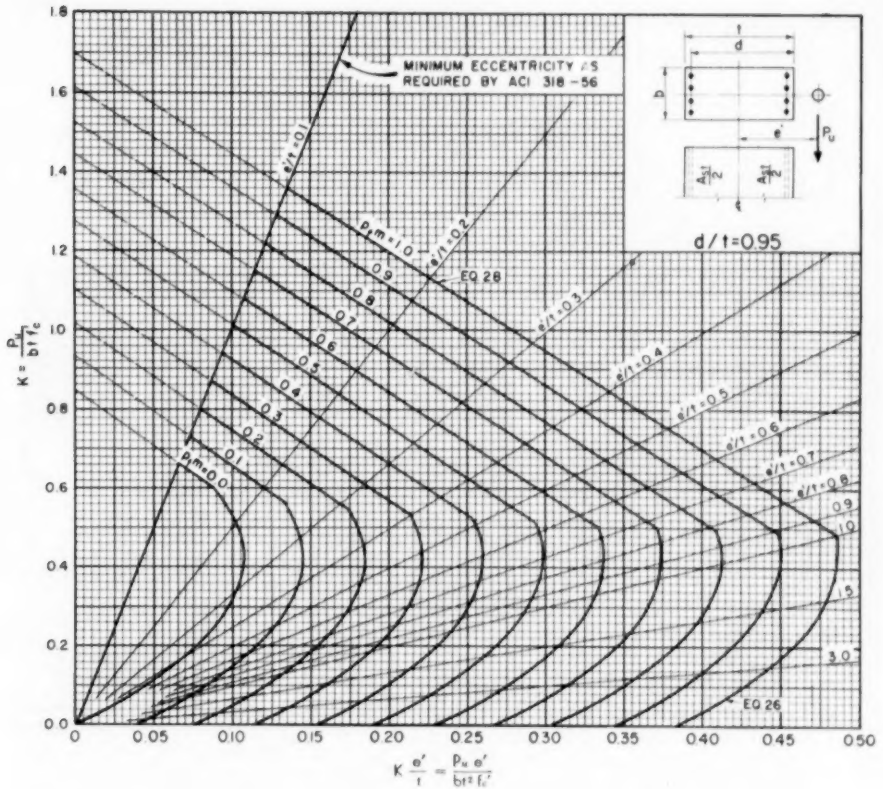

Chart 4—Bending and axial load—$d/t = 0.95$, rectangular sections with symmetrical reinforcement

Eq. (26)

$$P_u = 0.85\, f_c' bd\left\{1 - p - \frac{e}{d} + \sqrt{\left(1 - \frac{e}{d}\right)^2 + 2p\left[m'\left(1 - \frac{d'}{d}\right) + \frac{e}{d}\right]}\right\}$$

Eq. (28)

$$P_u = \frac{2A_s' f_y'}{\dfrac{2e'}{(d - d')} + 1} + \frac{btf_c'}{\dfrac{3te'}{d^2} + 1.18}$$

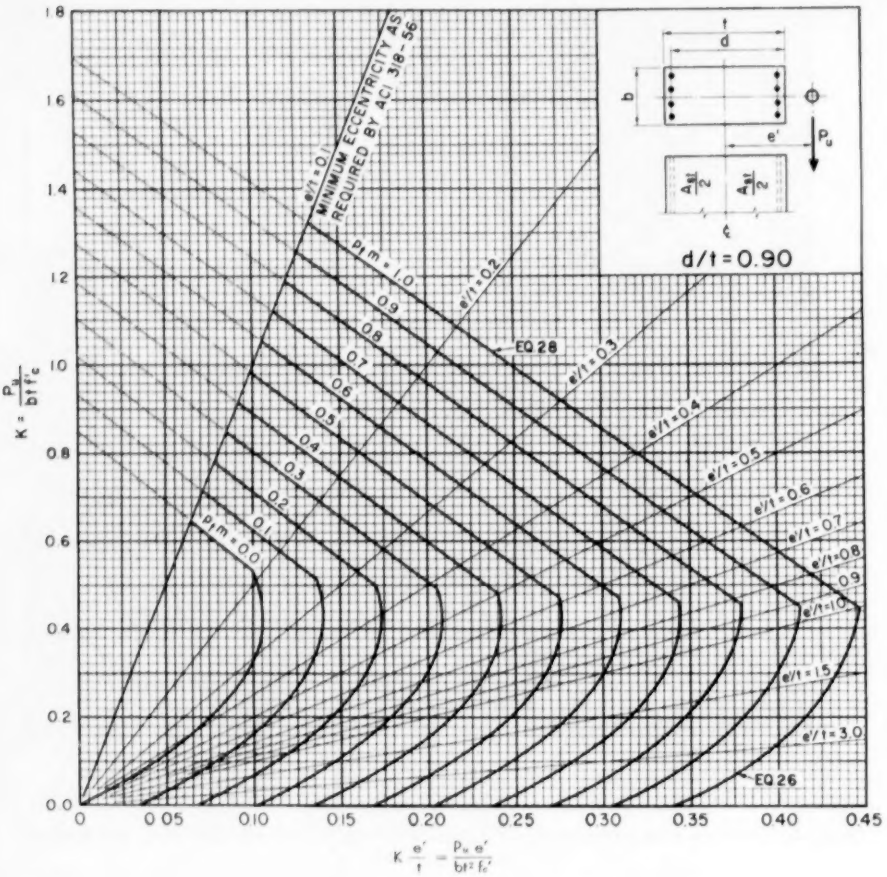

Chart 5—Bending and axial load—$d/t = 0.90$, rectangular sections with symmetrical reinforcement

Eq. (26)

$$P_u = 0.85\, f_c' bd\left\{1 - p - \frac{e}{d} + \sqrt{\left(1 - \frac{e}{d}\right)^2 + 2p\left[m'\left(1 - \frac{d'}{d}\right) + \frac{e}{d}\right]}\right\}$$

Eq. (28)

$$P_u = \frac{2A_s' f_y'}{\dfrac{2e'}{(d - d')} + 1} + \frac{btf_c'}{\dfrac{3te'}{d^2} + 1.18}$$

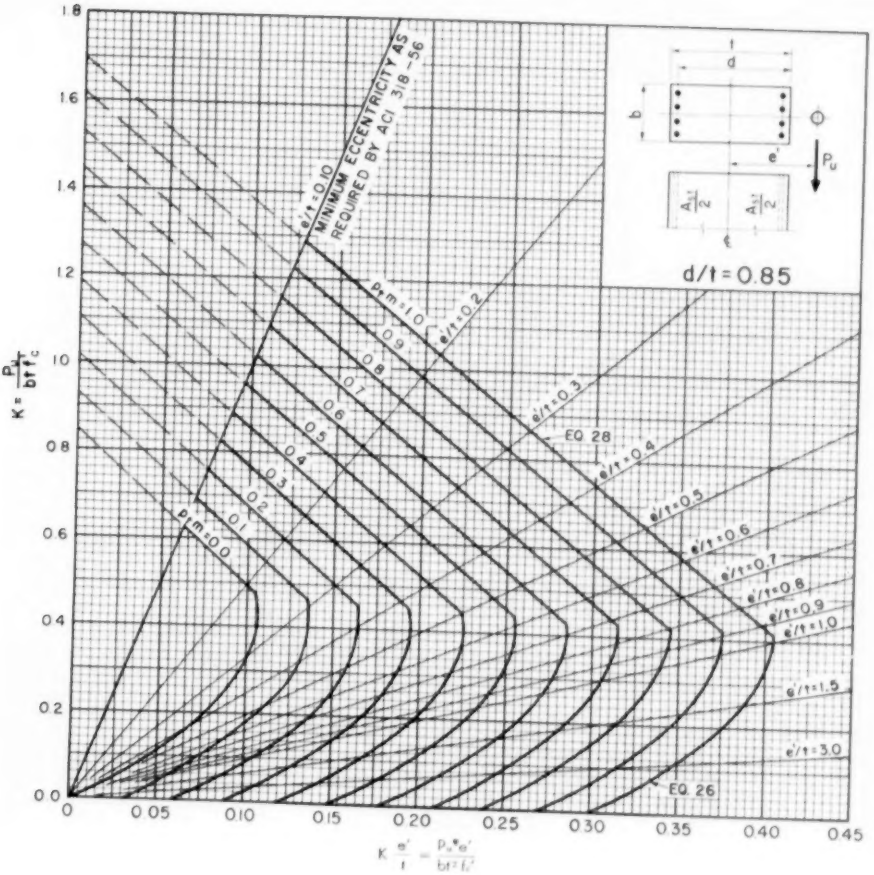

Chart 6—Bending and axial load—$d/t = 0.85$, rectangular sections with symmetrical reinforcement

Eq. (26)

$$P_u = 0.85 f_c' bd\left\{1 - p - \frac{e}{d} + \sqrt{\left(1 - \frac{e}{d}\right)^2 + 2p\left[m'\left(1 - \frac{d'}{d}\right) + \frac{e}{d}\right]}\right\}$$

Eq. (28) $$P_u = \frac{2A_s' f_y'}{\frac{2e'}{(d - d')} + 1} + \frac{btf_c'}{\frac{3te'}{d^2} + 1.18}$$

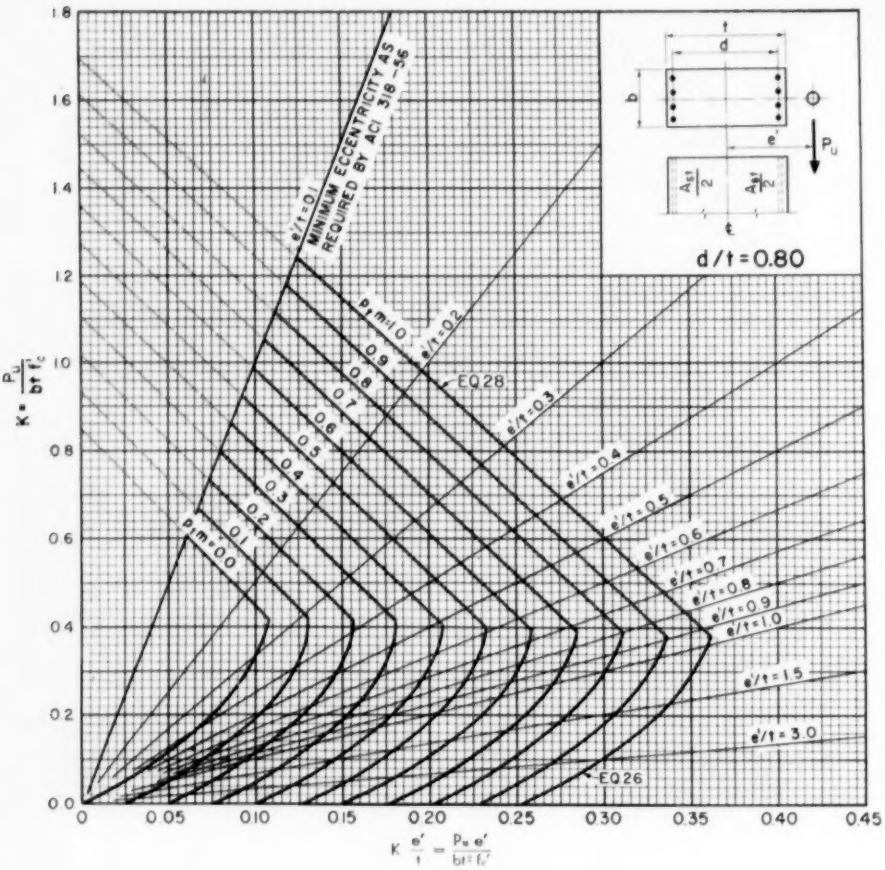

Chart 7—Bending and axial load—$d/t = 0.80$, rectangular sections with symmetrical reinforcement

Eq. (26)

$$P_u = 0.85\, f_c' bd\left\{1 - p - \frac{e}{d} + \sqrt{\left(1 - \frac{e}{d}\right)^2 + 2p\left[m'\left(1 - \frac{d'}{d}\right) + \frac{e}{d}\right]}\right\}$$

Eq. (28)

$$P_u = \frac{2A_s' f_y'}{\frac{2e'}{(d - d')} + 1} + \frac{btf_c'}{\frac{3te'}{d^2} + 1.18}$$

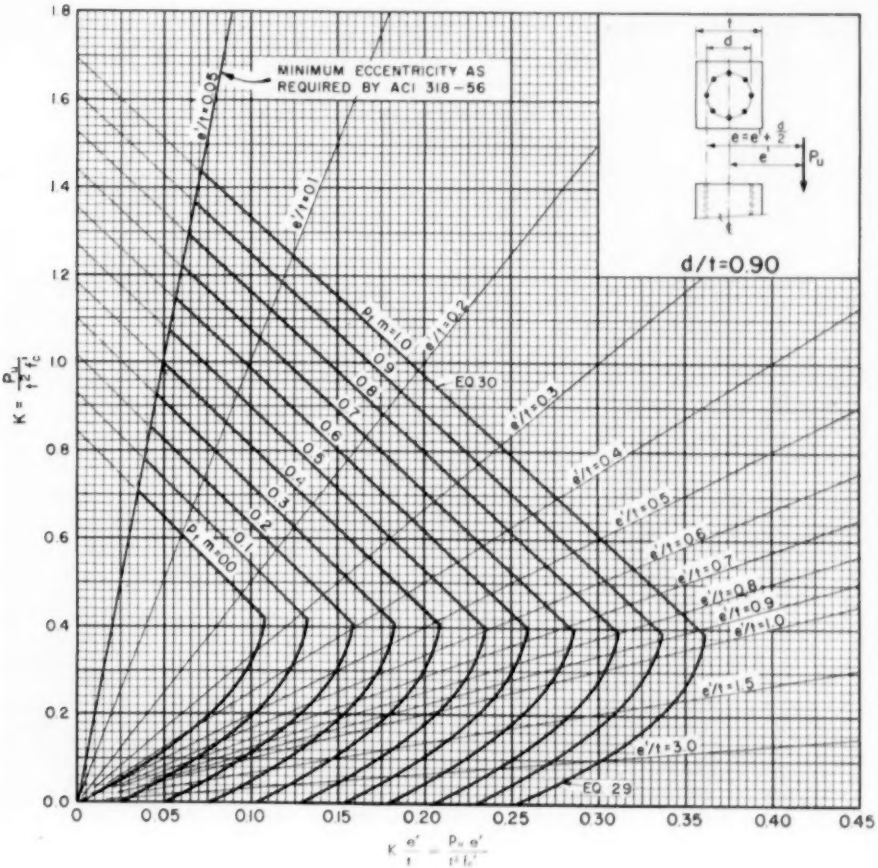

Chart 8—Bending and axial load—$d/t = 0.90$, square sections with spiral reinforcement

$$\textbf{Eq. (29)} \quad P_u = 0.85t^2 f_c' \left\{ \sqrt{\left(\frac{e'}{t} - 0.5\right)^2 + 0.67 \frac{d}{t} p_t m} - \left(\frac{e'}{t} - 0.5\right) \right\}$$

$$\textbf{Eq. (30)} \quad P_u = \frac{A_{st} f_y'}{\frac{3e'}{d} + 1} + \frac{A_g f_c'}{\frac{12.0t\, e'}{(t + 0.67d)^2} + 1.18}$$

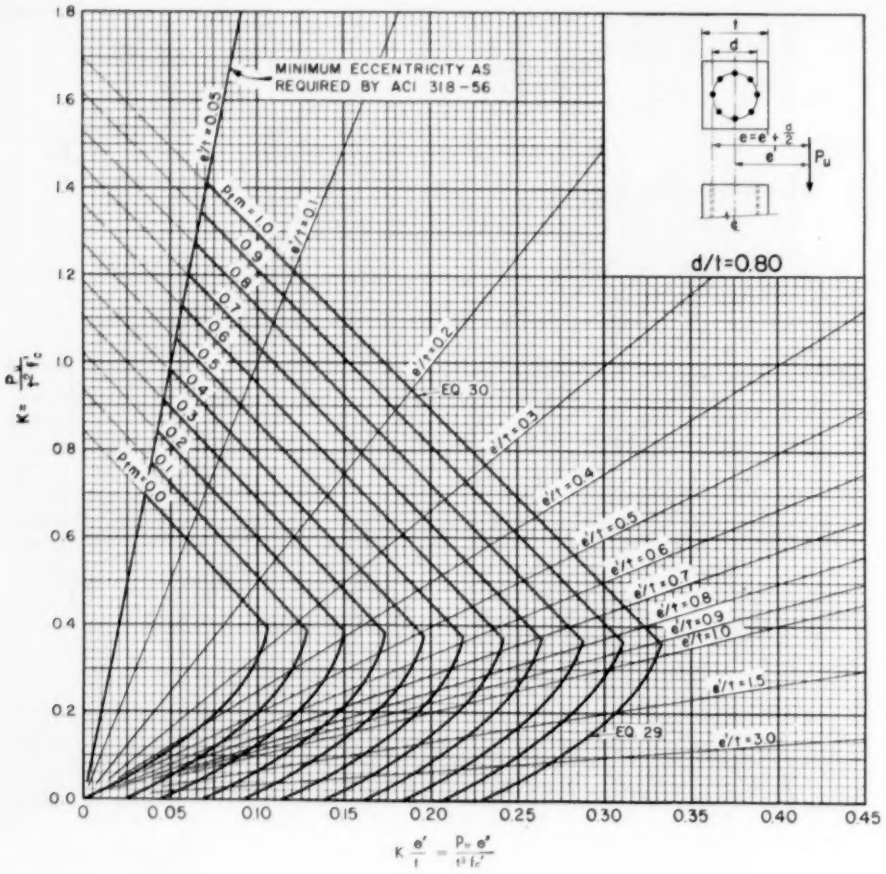

Chart 9—Bending and axial load—$d/t = 0.80$, square sections with spiral reinforcement

$$\textbf{Eq. (29)} \quad P_u = 0.85 t^2 f_c' \left\{ \sqrt{\left(\frac{e'}{t} - 0.5\right)^2 + 0.67 \frac{d}{t} p_t m} - \left(\frac{e'}{t} - 0.5\right) \right\}$$

$$\textbf{Eq. (30)} \quad P_u = \frac{A_{st} f_y'}{\frac{3e'}{d} + 1} + \frac{A_g f_c'}{\frac{12.0 t e'}{(t + 0.67d)^2} + 1.18}$$

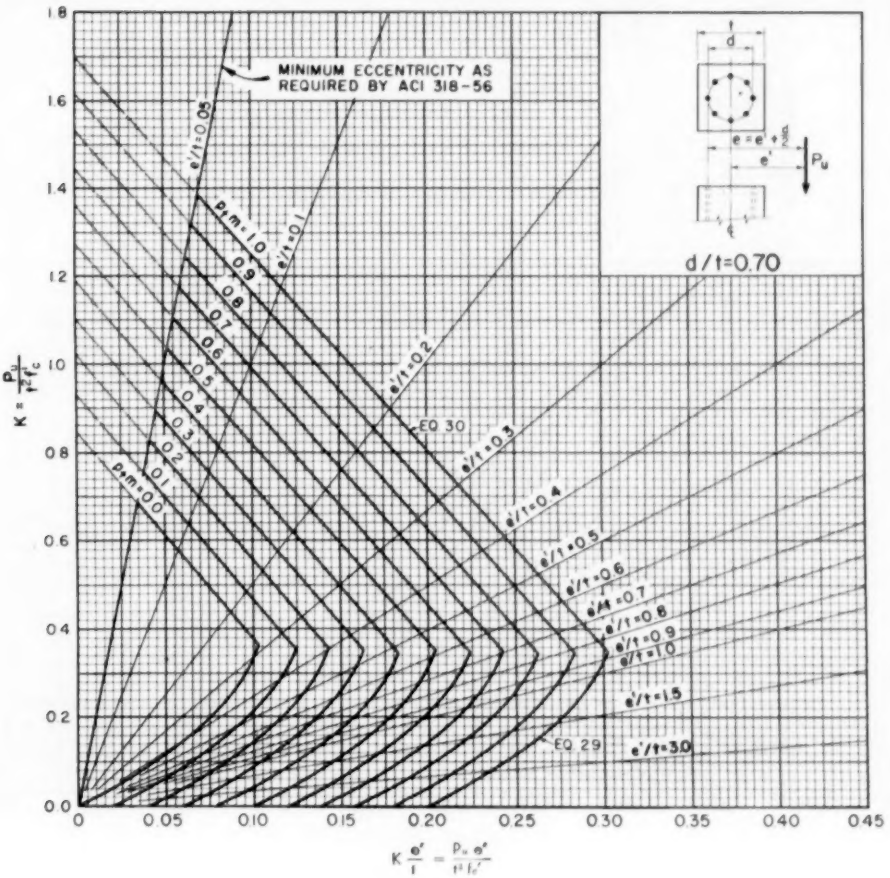

Chart 10—Bending and axial load—$d/t = 0.70$, square sections with spiral reinforcement

$$\textbf{Eq. (29)} \quad P_u = 0.85t^2 f_c' \left\{ \sqrt{\left(\frac{e'}{t} - 0.5\right)^2 + 0.67 \frac{d}{t} p_t m} - \left(\frac{e'}{t} - 0.5\right) \right\}$$

$$\textbf{Eq. (30)} \quad P_u = \frac{A_{st} f_y'}{\frac{3e'}{d} + 1} + \frac{A_g f_c'}{\frac{12.0t\,e'}{(t + 0.67d)^2} + 1.18}$$

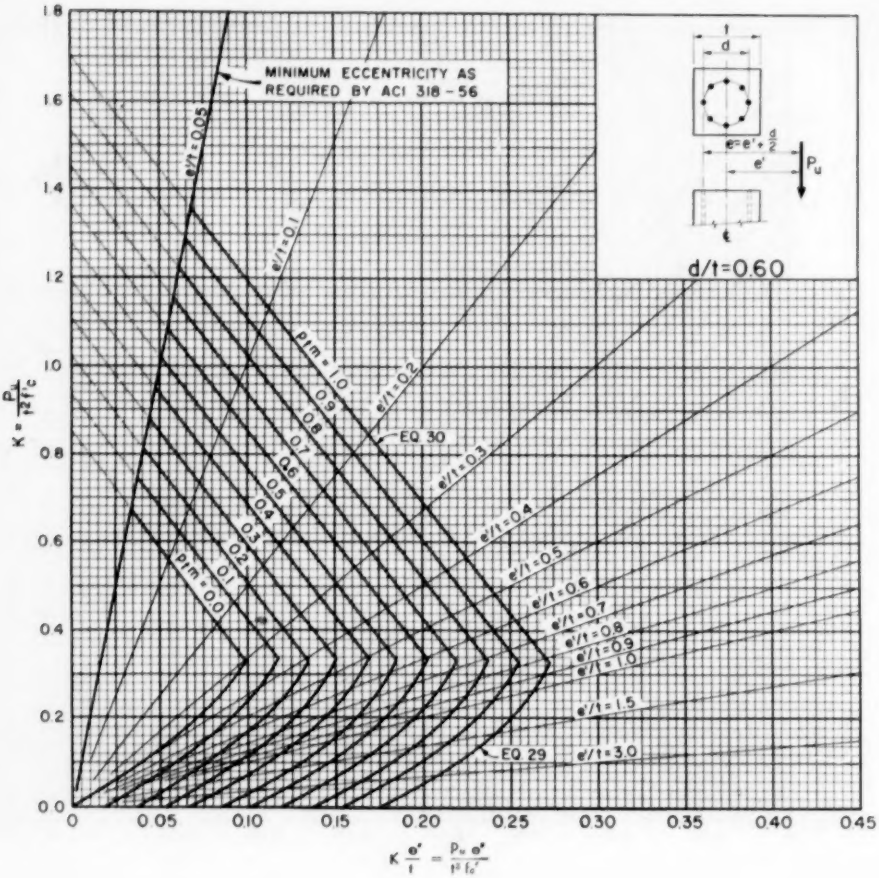

Chart 11—Bending and axial load—$d/t = 0.60$, square sections with spiral reinforcement

$$\textbf{Eq. (29)} \quad P_u = 0.85 t^2 f_c' \left\{ \sqrt{\left(\frac{e'}{t} - 0.5\right)^2 + 0.67 \frac{d}{t} p_t m} - \left(\frac{e'}{t} - 0.5\right) \right\}$$

$$\textbf{Eq. (30)} \quad P_u = \frac{A_{st} f_y'}{\frac{3e'}{d} + 1} + \frac{A_g f_c'}{\frac{12.0 t e'}{(t + 0.67d)^2} + 1.18}$$

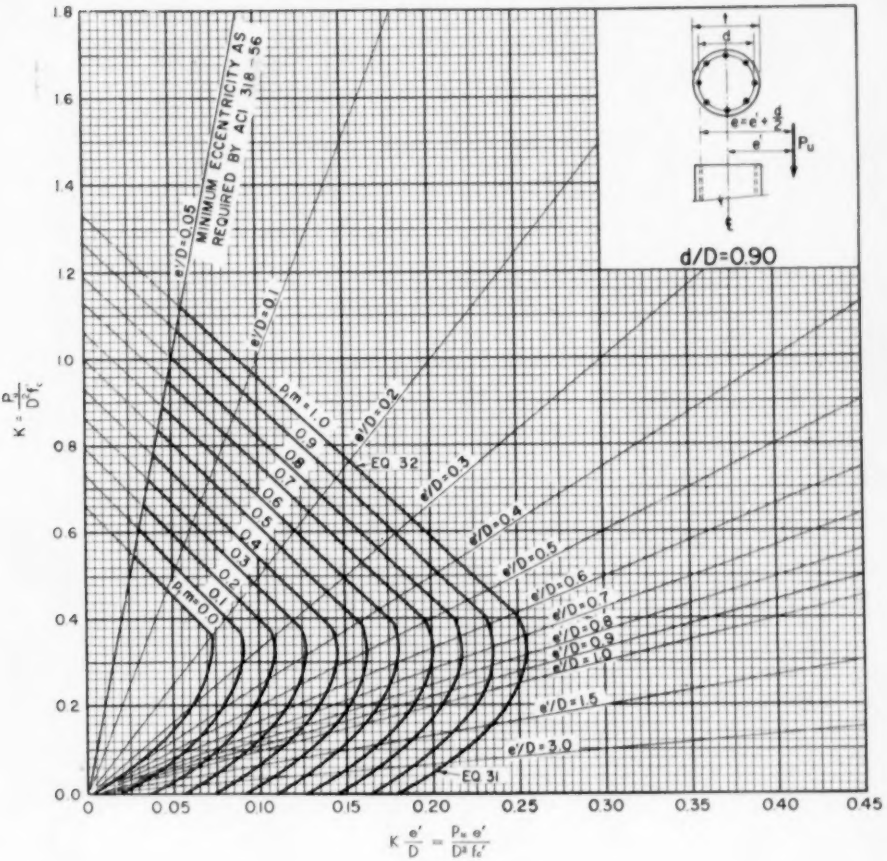

Chart 12—Bending and axial load—$d/D = 0.90$, circular sections with spiral reinforcement

Eq. (31)

$$P_u = 0.85D^2 f_c' \left\{ \sqrt{\left(\frac{0.85e'}{D} - 0.38\right)^2 + \frac{dp_t m}{2.5D}} - \left(\frac{0.85e'}{D} - 0.38\right) \right\}$$

Eq. (32)

$$P_u = \frac{A_{st} f_y'}{\dfrac{3e'}{d} + 1} + \frac{A_g f_c'}{\dfrac{9.6\,De'}{(0.8D + 0.67d)^2} + 1.18}$$

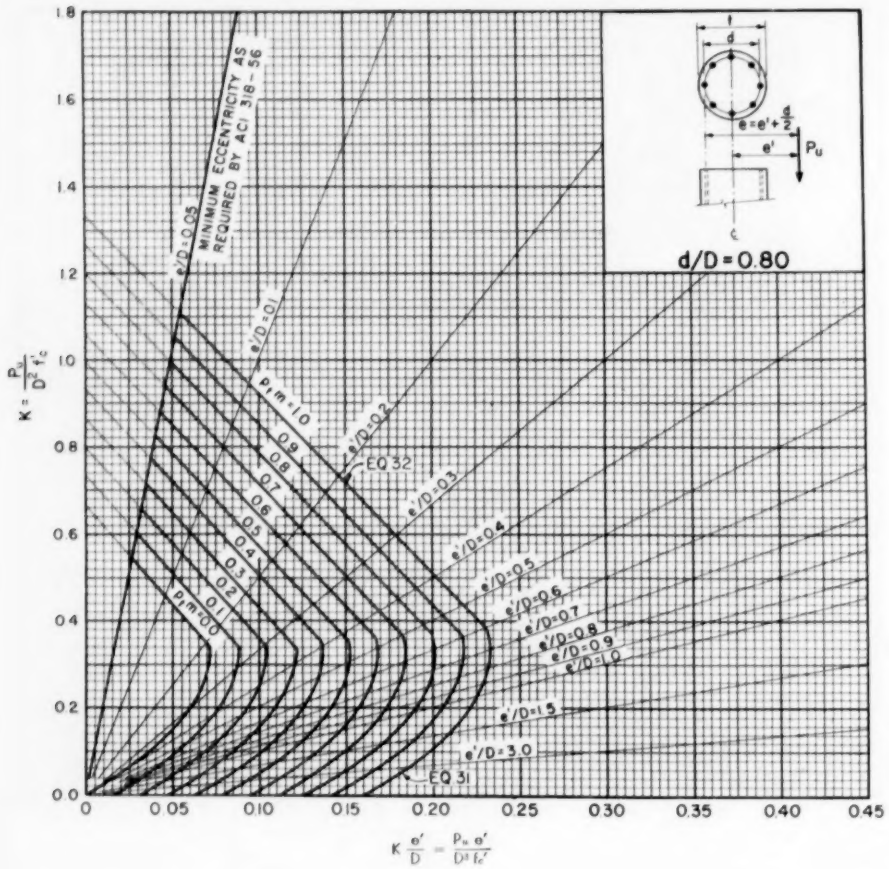

Chart 13—Bending and axial load—$d/D = 0.80$, circular sections with spiral reinforcement

Eq. (31)

$$P_u = 0.85D^2 f_c' \left\{ \sqrt{\left(\frac{0.85e'}{D} - 0.38\right)^2 + \frac{dp_g m}{2.5D}} - \left(\frac{0.85e'}{D} - 0.38\right) \right\}$$

Eq. (32)

$$P_u = \frac{A_s f_y'}{\frac{3e'}{d} + 1} + \frac{A_g f_c'}{\frac{9.6\,De'}{(0.8D + 0.67d)^2} + 1.18}$$

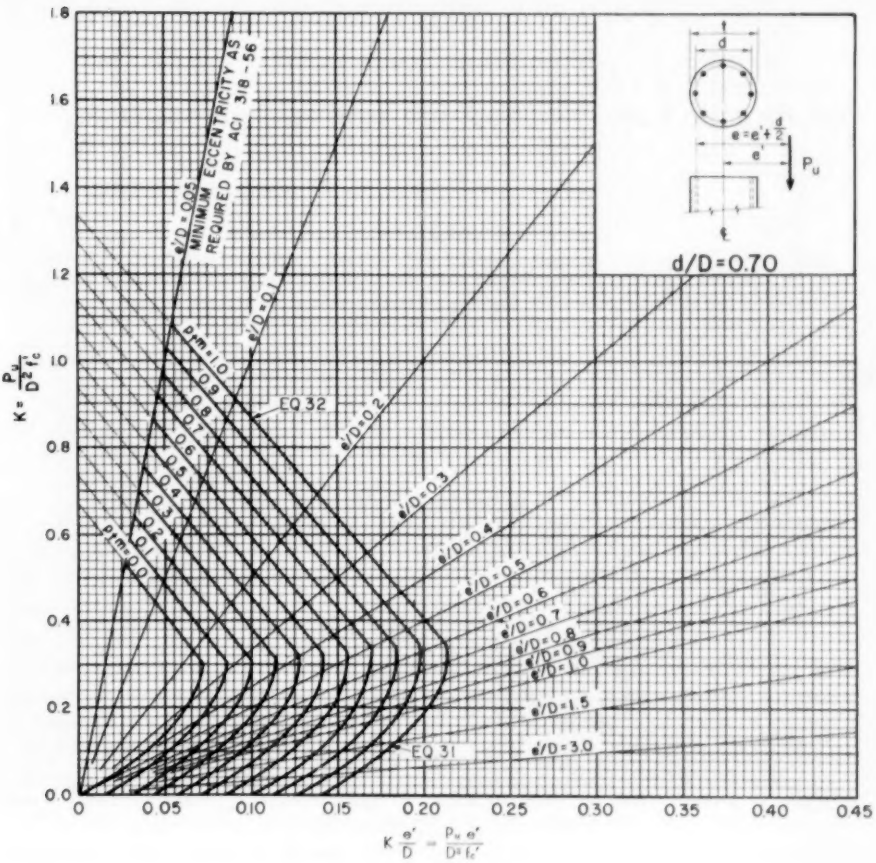

Chart 14—Bending and axial load—$d/D = 0.70$, circular sections with spiral reinforcement

Eq. (31)

$$P_u = 0.85D^2 f_c' \left\{ \sqrt{\left(\frac{0.85e'}{D} - 0.38\right)^2 + \frac{dp_t m}{2.5D}} - \left(\frac{0.85e'}{D} - 0.38\right) \right\}$$

Eq. (32)

$$P_u = \frac{A_{st} f_y'}{\frac{3e'}{d} + 1} + \frac{A_g f_c'}{\frac{9.6\,De'}{(0.8D + 0.67d)^2} + 1.18}$$

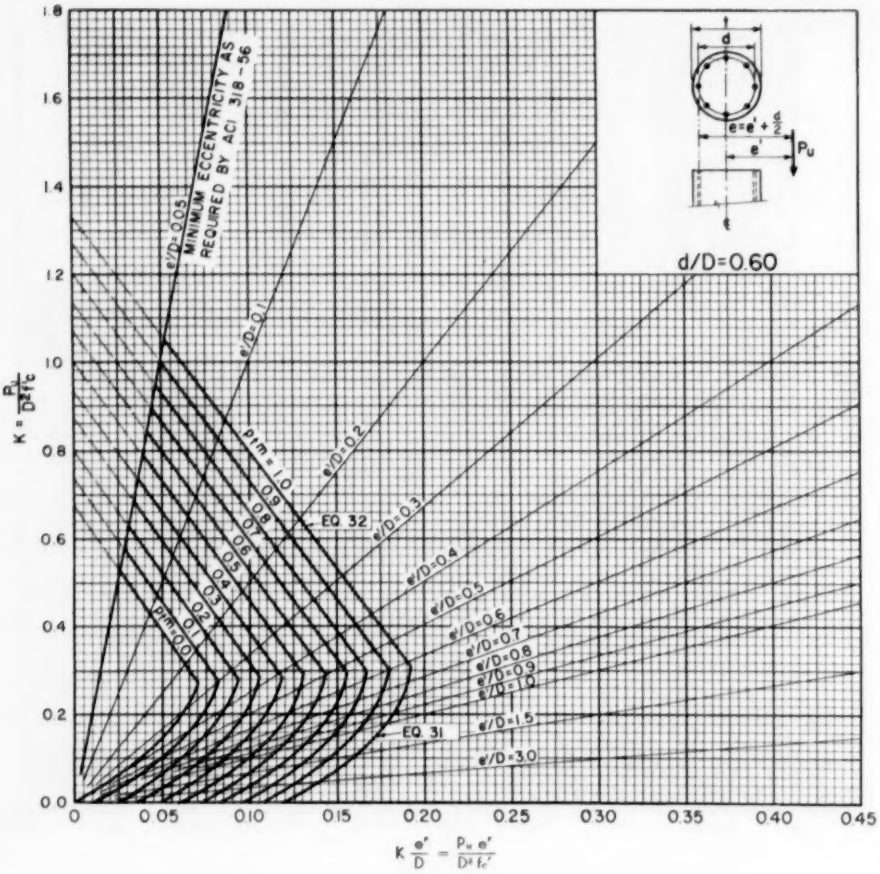

Chart 15—Bending and axial loads—$d/D = 0.60$, circular sections with spiral reinforcement

Eq. (31)

$$P_u = 0.85D^2 f_c' \left\{ \sqrt{\left(\frac{0.85e'}{D} - 0.38\right)^2 + \frac{dp_t m}{2.5D}} - \left(\frac{0.85e'}{D} - 0.38\right) \right\}$$

Eq. (32)

$$P_u = \frac{A_{st} f_y'}{\frac{3e'}{d} + 1} + \frac{A_g f_c'}{\frac{9.6\,De'}{(0.8D + 0.67d)^2} + 1.18}$$

Enlargements of these design charts (approximately 12 x 12 in.) are available in sets only at $2.00 per set of 15 charts. Order from ACI headquarters.

Title No. 53-26

The third paper in a series of four on lightweight concrete

Lightweight Aggregates for Concrete Masonry Units*

By C. C. CARLSON†

SYNOPSIS

The role of mineral lightweight aggregates in the production and utilization of concrete masonry units is discussed. Differences between lightweight and heavyweight aggregates having influence in block manufacturing procedure are cited and the means for accommodating them are given. Various physical properties of lightweight aggregate concrete masonry wall construction are described and compared to those of heavyweight aggregate. Some mention is made of special uses of lightweight aggregate masonry units for precast and cast-in-place floor and roof construction.

INTRODUCTION

This paper discusses mineral lightweight aggregates commercially utilized for the production of machine-molded masonry units and allied products from dry-mix or dry-tamp‡ concrete. These aggregates conform to the requirements of ASTM C 331-53T. The more commonly used types include: expanded and processed blast furnace slag, clay, shale, slate, processed pumice, scoria, tuff, and processed cinders from coal burning industrial furnaces. The concretes employed generally range in compressive strength from about 1200 to 3000 psi, net area, although higher strength concretes are obtainable with the stronger lightweight aggregates.

The demand for lightweight aggregate for concrete masonry units derives, in part, from a desire to reduce the weight of masonry units and of structures built thereof. The modular 8 x 8 x 16-in. hollow load-bearing air-dry concrete block in the current two- and three-cell designs, composed of ordinary aggregates, such as sand and gravel or crushed stone, weighs approximately 45 lb. Its counterpart of lightweight aggregate averages about 30 lb and may weigh as little as 22 lb. Lightweight aggregates impart other properties to concrete masonry units and masonry construction that are perhaps of greater importance in accounting for their demand. Some of these are increased thermal resistance, sound absorption, fire resistance, and nailability. Some lightweight aggregate types contribute certain color shading and

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†Member American Concrete Institute, Manager, Fire Research Section, Portland Cement Association, Chicago, Ill.

‡A term descriptive of block concrete consistency which has been carried over from an earlier period of the block industry when molding was accomplished in power tamping machines.

textural effects to concrete masonry construction that have been utilized for the interior treatment of auditoriums, school rooms, churches, office buildings, hospitals, and private dwellings.

GENERAL CONSIDERATIONS

Aside from certain economic factors, such as cost and sustained availability, manufacturers of concrete masonry units using lightweight aggregates are concerned with various properties of these aggregates and the influence of the properties on the finished product. Probably the most important properties are unit weight, grading, and strength. They determine to a considerable extent the weight of the product and the cement requirements to meet given strength stipulations. The user should not overlook other properties affecting final results, such as color, absorption, freedom from deleterious substances, particle shape, and particle surface characteristics.

Unit weight of lightweight aggregate

ASTM C 331-53T limits the unit weight of delivered lightweight aggregate to the following maximum values:

Size classification	Dry loose weight,* maximum, lb per cu ft
Fine aggregate	70
Coarse aggregate	55
Combined fine and coarse aggregate	65

*See ASTM C 29-42, Section 7, for details on determination of dry loose unit weight.

The dry loose unit weight of graded lightweight aggregates available for concrete masonry units varies considerably, ranging from 40 to 70 lb per cu ft. Highly clinkered coal cinder aggregates tend to the higher value. Within the size groups of a given lightweight aggregate it is important to good control of masonry unit quality that a reasonable sameness of unit weight be maintained in successive shipments of that aggregate. The above ASTM standard limits the variation in unit weight to 10 percent from that of the sample submitted for acceptance tests.

Strength of lightweight aggregate

It appears that the best gage of aggregate strength is its performance, strength-wise, in concrete. Strength comparisons in concrete should, of course, be made with all other factors equal, such as cement content, consolidation, curing, and age and moisture content at testing. Some effort has been directed to the development of a rapid test[2] for predicting the strength property of lightweight aggregate in concrete. The test involves the measurement of the crushing resistance of graded aggregate when loaded in a hollow cylinder. Only general correlation has been found between the crushing strength of aggregate exhibited by this test and compressive strength of the concrete made from the aggregate.

Another aspect of the strength of lightweight aggregate that has bearing on the properties of the finished concrete product is the resistance of aggregate to attrition and breakage during the mixing operation or other handling. A few types, a small minority, tend to friability and the coarse particles, particularly, may break down to produce a grading and concrete weight different from what may have been sought.

Grading

ASTM C 331-53T (Table 1 of standard) permits considerable latitude in the proportions of the different sizes for acceptable grading of lightweight aggregates for concrete masonry units. However, this should not be interpreted as opportunity for an easy-going approach to the problem of aggregate grading. Rather, the indicated latitude is necessary to accommodate the fairly wide differences in particle shape and texture that exist between the different types. Thus, what may be proper grading for one type may not apply for another type.

The matter of grading is of particular importance and interest in the manufacture of lightweight concrete masonry units because a number of physical qualities sought in the product are affected by it, such as the strength and weight of the masonry unit, the texture of the exposed surfaces, and the heat insulating and sound absorbing properties. Not to be overlooked are the production aspects of aggregate grading. Grading affects the moldability and cohesiveness of the concrete and the resistance of freshly molded units to cracking or breakage in handling.

Producers of lightweight aggregates frequently ship their material in two size groups, coarse and fine. The dividing point is usually on the No. 4 screen or in its vicinity. This practice is of assistance in the problem of segregation which is probably more aggravated in the case of lightweight aggregates than in the case of the heavyweight types because of the greater difference in bulk specific gravity between the coarse and fine sizes. When aggregate is received in this manner, the consumer should, of course, be equipped with separate bins or divided bins to accommodate the two sizes. With such equipment he is in a position to exercise a degree of control over the grading that obtains finally in the mixer, and to adjust grading as textural or other physical property requirements in the manufactured product direct.

Table 1 contains gradings of coarse and fine size groupings of expanded shale and slag aggregate that have been successfully used in the commercial production of concrete masonry units.

How the coarse and fine size groups of an aggregate should be combined to provide the best grading for a particular operation is a question which may not be easily answered and considerable study and experimentation may be required before the desired grading is realized. Some data reported by Menzel[15] on tests of hollow Haydite block made from concrete mixes containing similar amounts of cement but with different aggregate gradings show that the compressive strength of the block (based on the gross area) decreased

TABLE 1—TYPICAL MECHANICAL ANALYSIS OF LIGHTWEIGHT AGGREGATES FOR CONCRETE MASONRY UNITS[3]

Aggregate size group	Screen size, percentages (by weight) passing							
	⅜ in.	No. 4	No. 8	No. 16	No. 30	No. 50	No. 100	F.M.
Expanded shale								
Fine	—	100	87	57	39	31	21	2.65
Coarse	95	30	4	3	3	3	2	5.60
Expanded slag								
Fine	—	97	76	55	39	25	16	2.92
Coarse	92	27	15	13	13	10	7	5.23

rather rapidly as the fineness modulus increased beyond about 3.75. The optimum fineness modulus with respect to the strength of the block of this particular lightweight aggregate was about 3.25. At this point the weight of the hardened and air-dry unit was approximately maximum. Companion data on similar hollow-block containing different heavy aggregate types show that each of these heavy aggregates had its optimum fineness modulus where strength and weight of the block were maximum. When the fineness modulus was varied in either direction from these optima, strength and weight dropped off sharply.

Aggregate grading for concrete block mixes is discussed in NCMA *Bulletin* No. 5[4] and suggested gradings based on bulk volume are given for three block textures—coarse, medium and fine. These gradings were obtained by plotting selected cumulative percentages of material retained on the different screens to produce reasonably smooth curves with desired amounts of material coarser than the No. 4, smaller than the No. 50, and with the desired fineness modulus. The volumetric percentage values are directly applicable for grading on the weight basis if the bulk specific gravities of the different sizes of aggregate are approximately the same. However, where differences in bulk specific gravity between the different sizes are encountered (in some lightweight aggregate types) a correction must be applied. A method for making this correction is illustrated. Examples are also given which show how desired gradings can be obtained from combinations of coarse and fine size groups of aggregate meeting the ASTM requirements.

Deleterious substances

Various deleterious substances can occur in lightweight aggregates and various tests are described in the ASTM specifications for their detection and limitation. Some of the better known contaminants peculiar to the different lightweight aggregates are: iron and iron compounds, hard-burned lime, clay lumps, and unburned or partially burned coal. Iron contamination is encountered principally in coal cinders. Iron and some compounds thereof produce unsightly staining on exposed masonry surfaces and some compounds

of iron cause popping. A method for detecting staining materials in lightweight aggregates is described under Section 7(i) of ASTM C 331-53T. Seaton[21] has discussed methods for treating cinder aggregates to minimize staining due to contained iron compounds. These methods appear applicable to other lightweight types where staining derives from iron and iron compounds. Hard-burned lime contamination obtains from the reduction of occluded limestone particles to calcium or magnesium oxide in heat treating or heat producing processes, such as the rotary kiln, sintering grate, or coal burning industrial furnace, and is more commonly manifested by pop-outs in masonry wall surfaces. Pop-outs arising from this contaminant may be delayed for a considerable time after the wall construction has been placed in service and may continue for an unpredictable period. The autoclave test [Section 8(b) of ASTM C 331-53T] provides a means for its detection. Sizable lime occlusions can cause cracking and disruption of the concrete. Clay contamination is more often associated with natural lightweight aggregates, such as scoria and pumice, which derive from bedded deposits. Unburned or partially burned coal which tends to air-slake may cause pop-outs and if present in excessive quantity can lead to destructive expansion of the concrete.

PRODUCTION OF CONCRETE MASONRY UNITS OF LIGHTWEIGHT AGGREGATE

Proportioning and mixing

In general, the procedures for proportioning and mixing heavyweight and lightweight aggregate concretes in block manufacture are the same. There are some differences in details, however, which arise from the differences in aggregate unit weight, particle characteristics, and water absorption. Most concrete products plants are equipped with traveling batchers operated in conjunction with overhead supply bins or silos for proportioning concrete materials. The batchers are usually designed so that they may be used for either weight or volume proportioning, as the producer may choose. Weight batching is probably in greater use than the volumetric method. Some users of lightweight aggregates for concrete block prefer to proportion aggregate on a volumetric basis because of the lesser effect of moisture variations. Either method requires watchfulness of variations in moisture content as well as unit weight in delivered aggregate so that the established grading and yield and strength of block are maintained.

The most widely used mixer is the fixed-drum revolving-blade batch mixer. This type ranges to 80 cu ft in capacity and is evidently well adapted for preparing dry-mix lightweight aggregate concrete. There is rather general agreement in the industry, based on plant experience rather than on any extensive test data, that lightweight aggregates need to be prewetted in the mixer before the cement is introduced. This practice is said to serve the dual purpose of satisfying the immediate absorptivity of the aggregate and of preventing cement from being drawn into large voids and interstices of rough-surfaced aggregate particles where it is presumed to be ineffective and to add

useless weight. Prewetting is accomplished by mixing the aggregate and ½ to ⅔ of the total estimated water for 1 to 2 min before the cement is added. Tests conducted by the National Concrete Masonry Association,[7] using 50 cu ft capacity mixers of conventional design, indicate that the mixing time, including the prewetting period for lightweight aggregates, should not be less than 5 min. Good practice requires that a timing device be provided at the mixer so that full mixing time for each batch of concrete is assured.

The use of air-entraining cement or air-entraining additives in preparing lightweight aggregate concretes for masonry unit production is increasing. Entrained air is said to improve concrete moldability and cohesion of freshly molded units, particularly where aggregate particles are angular and rough-surfaced. It also improves concrete durability.* When air entrainment is directed to improving workability, experience has shown that lightweight aggregates vary in their requirements of air-entraining agent to produce desired results. Aggregates consisting of smooth-surfaced and rounded particles require less air-entraining agent than those of which the particles are rough and angular.

Excessive inter-particle voids will be present in concrete block, even if produced from an aggregate of optimum grading, if the mix is too dry.[4] Usually, excessively dry mixes are more difficult to compact, give higher block porosity, poorer green block stability (cohesion), and lower compressive strength. For a particular block mix the maximum concrete density (minimum voids) and strength are obtained by using as much mixing water as is possible short of slumping or loss of desired open face shell texture (smearing). Generally, the maximum water content will be determined by the tendency to produce smeared surfaces on freshly molded units. In recent years various devices have been developed to control water in the concrete mix at predetermined amounts. One type depends for its operation on the change in electrical resistance that occurs in the mixer charge as water is added.

Molding

Vibration (with pressure) molding machines have largely supplanted the tamper-type during the past 15 years. This change has been brought about by the greater efficiency and productive capacity of the vibration machine and other advantages.[3,5] Vibration machines evidently are adaptable to a wider range of lightweight aggregate types because of the minimum pressure exerted on the concrete during consolidation in contrast to the heavy impact action of the tamper machines. Some pressure (about 7 psi) with vibration consolidation appears essential in molding lightweight aggregate concrete for maximum strength development of block. Presumably, this need derives from the relatively low workability of dry-mix concrete and the short vibration period (about 12 sec†) permitted in modern block production.

*From unpublished data of Research and Development Division, Portland Cement Association.
†Based on a production rate of 750–8 x 8 x 16-in. hollow block per hour, three units per cycle.

Curing

Curing practices are in keeping with the accelerated tempo of companion operations in concrete masonry unit production. In the majority of plants, hardening is expedited by exposing the units to moist air in kilns at temperatures ranging to 200 F. This curing procedure is commonly known as steam curing at atmospheric pressure. Much has been written concerning high-temperature moist-air curing (maximum kiln temperatures ranging from 140 to 200 F) and considerable discussion still prevails as to what is the best formula. The majority of plants employ maximum curing temperatures ranging from 145 to 160 F for heavyweight aggregate block and 165 to 180 F for lightweight aggregate block. Curing durations for both heavy and lightweight aggregate concretes after maximum curing temperature is reached are about the same—12 to 15 hr.

A small but growing proportion of plants employ autoclaving procedures for conditioning concrete masonry units. The literature indicates that current autoclaving practice is about the same for lightweight units as for heavyweight units but varies with respect to maximum steam pressures and curing duration. It appears that the benefits of high-pressure steam curing may be fully realized if sufficient finely divided siliceous material is available in the mix and if the curing cycle includes the following:

(a) A waiting period after molding of at least 2 hr at normal temperature before steam is applied.

(b) A heating period of at least 3 hr during which the saturated steam temperature (and pressure) is increased uniformly to the desired level.

(c) A period during which saturated steam temperature is maintained for at least 8 hr at 350 F (120 psi gage) or for 7 hr at 365 F (150 psi gage).

(d) A reduction of autoclave pressure after curing to atmospheric pressure within 30 min.

Menzel[6] recommends 5 hr as a suitable waiting period but points out that Haydite block are affected to a lesser degree than sand and gravel by a reduced waiting period. His data indicate that for the leaner mixes, such as are in current use, a waiting period of 2 hr can be tolerated by block of either type of aggregate without undue sacrifice of strength gain.

Drying

Current ASTM and federal specifications covering delivered units require that the moisture content shall not exceed 40 percent of the total water absorption. More recently promulgated guide specifications of the Corps of Engineers on concrete masonry for military construction, CE-206.01, limit the maximum moisture content of masonry units to 30 percent of the total absorption at the time of delivery.* Concrete masonry units must therefore be dried subsequent to curing. Some producers that employ steam curing at atmospheric pressure dry their product either naturally in ventilated stock piles under roof protection or use artificial methods requiring heated air. Concrete masonry units of lightweight aggregate vary considerably with

*See also reference 20, p. 3.

the aggregate type in their response to natural as well as artificial drying. In general, lightweight aggregates having a rather open pore structure, such as expanded slag and cinders, produce block that respond quite readily to air drying. Units containing aggregate characterized by high absorption and finely divided pore structure, such as pumice, are more reluctant in their release of moisture and may require artificial drying periods of considerable duration. Air drying of such units is accomplished more practicably, perhaps, by the slower natural method.

In autoclave curing the evaporation that occurs in the "blow-off" or pressure reduction phase and subsequent cooling from temperatures in the range of 350 to 365 F is usually sufficient to reduce the moisture content of masonry units of most lightweight aggregate types to required limits. However, the "blow-off" must be accomplished in a comparatively short time (not more than 30 min) to achieve this result. Vacuum drying at 20 or more inches of mercury, subsequent to the "blow off" phase, has been used with good effect on lightweight aggregate types that are particularly resistive.

PHYSICAL PROPERTIES OF LIGHTWEIGHT CONCRETE MASONRY UNITS AND MASONRY WALL CONSTRUCTION

Strength

Building codes and specifications covering concrete masonry building units identify the various types as:

1. Hollow load-bearing concrete block
2. Solid load-bearing concrete block
3. Hollow non-load bearing-concrete block
4. Concrete building tile
5. Concrete brick

Minimum compressive strength requirements are given for each type and grade. These masonry units are regularly made with both lightweight and heavyweight aggregates. Table 2 summarizes the minimum requirements of the federal* and ASTM specifications with respect to compressive strength and limitations on absorption and moisture content. Generally the codes and specifications do not differentiate between aggregate types in their requirements on strength. Presumably, this generalization springs from a practical need for code simplification. The given minimum compressive strengths of the different masonry unit types and the corresponding allowable working compressive stresses in built-up masonry walls are such that differentiation between aggregate types is unnecessary (Table 3).

The block producer has found through experience that the minimum requirements for compressive strength of masonry units cannot be used as the sole basis for concrete design. Other factors, such as resistance to breakage in handling, sharply defined and durable corners, resistance to cracking in masonry, durability of the masonry in service, fire resistance, nailability, sound absorption, and resistance to sound transmission are often more im-

*Does not include the requirements of Corps of Engineers' specifications, CE-206.01, which apply for military construction.

TABLE 2—SUMMARY OF MINIMUM PHYSICAL REQUIREMENTS FOR VARIOUS TYPES OF CONCRETE MASONRY UNITS[10]

Specification, serial designation, and latest revised date	Minimum face-shell thickness, in.	Compressive strength, minimum, psi, average gross area		Water absorption, maximum, lb per cu ft of concrete, average of 5 units	Moisture content, maximum, percent of total absorption, average of 5 units
		Average of 5 units	Individual unit		
Hollow load-bearing concrete masonry units ASTM C 90, 1952	1¼ or over:				
Grade A*		1000	800	15	40
Grade B†		700	600	—	40
Under 1¼ and over ¾		1000	800	15	40
Hollow non-load-bearing concrete masonry units ASTM C 129, 1952	Not less than ½	350	300	—	40
Solid load-bearing concrete masonry units ASTM C 145, 1952‡					
Grade A	—	1800	1600	15	40
Grade B	—	1200	1000	15	40
Concrete units; masonry hollow Federal SS-C-621, 1935				Average of 3 units	Average of 3 units
Load-bearing units	1¼ or more	700	600	16	40
	¾ to 1¼	1000	800	16	40
Non-load-bearing units	Not less than ¾	350	—	—	40
Concrete building brick ASTM C 55, 1952		(brick flatwise)			
Grade A	—	2500	2000	15	40
Grade B	—	1500	1250	—	40

*For use in exterior walls below grade, and for unprotected exterior walls above grade.
†For general use above grade where protected from the weather with two coats of portland cement paint or other satisfactory waterproofing treatment approved by the purchaser.
‡Units with 75 percent or more net area. The classification is based on strength and does not necessarily measure weather resistance.

portant. Thus, he designs for specific properties but more generally endeavors to strike a balance which satisfies the strength stipulations and the maximum number of the other considerations.

Except for special applications—lintels, reinforced structural floor and roof elements, or heavily loaded piers and columns—the block manufacturer will usually employ one concrete mix which provides the best all around product performance with the particular aggregate he chooses to use. Ac-

TABLE 3—ALLOWABLE COMPRESSIVE LOADING, CONCRETE MASONRY WALLS[20]

Kinds of masonry and of concrete masonry units	Allowable compressive stress, psi, gross area of masonry		
	Type of mortar*		
	A_1	A_2	B
Masonry of hollow load-bearing concrete masonry units	85	75	70
Solid masonry of concrete brick and of solid load-bearing concrete masonry units			
Grade A	175	160	140
Grade B	125	115	100

*ASTM Designation C 270-52T. Average compressive strength of 2-in. cubes at 28 days should be: Type A_1 mortar—2500 psi, Type A_2—1800 psi, Type B—750 psi.

TABLE 4—WEIGHTS OF CONCRETE MASONRY WALLS CONSTRUCTED OF HEAVYWEIGHT AND LIGHTWEIGHT AGGREGATE MASONRY UNITS[10]

Actual unit sizes (width x height x length), in.	Nominal wall thickness, in.	For 100 sq ft of wall					
		Number of units	Average weight of finished wall		Mortar,‡ cu ft	Weight savings	
			Heavyweight aggregate, lb*	Lightweight aggregate, lb†		lb	Percent
3⅝ x 3⅝ x 15⅝	4	225	3050	2150	4.3	900	29.5
5⅝ x 3⅝ x 15⅝	6	225	4550	3050	4.3	1500	33.0
7⅝ x 3⅝ x 15⅝	8	225	5700	3700	4.3	2000	35.0
3¾ x 5 x 11¾	4	221	3000	2150	3.7	850	28.3
5¾ x 5 x 11¾	6	221	4500	3050	3.7	1450	32.3
7¾ x 5 x 11¾	8	221	5650	3700	3.7	1950	34.5
3⅝ x 7⅝ x 15⅝	4	112.5	2850	2050	2.6	800	28.0
5⅝ x 7⅝ x 15⅝	6	112.5	4350	2950	2.6	1400	32.3
7⅝ x 7⅝ x 15⅝	8	112.5	5500	3600	2.6	1900	34.6
11⅝ x 7⅝ x 15⅝	12	112.5	7950	4900	2.6	3050	38.4

Table based on ⅜-in mortar joints.
*Actual weight within ± 7 percent of average weight.
†Actual weight within ± 17 percent of average weight.
‡With face-shell mortar bedding. Mortar quantities include 10 percent allowance for waste—droppings within the core space and extrusion beyond the bearing areas of units.

cording to a survey of the industry by a European observer,[11] no producer follows a studied practice of making non-load-bearing masonry units with the available structural lightweight aggregates in compressive strengths less than 600 psi (gross area), although the specifications permit 350 psi. Units with strengths lower than 600 psi apparently have been found too difficult and costly (culling losses) to manufacture.

Weight

Table 4 provides an insight to the weight savings in concrete masonry wall construction possible through the use of lightweight aggregates. The wall weights given are based on averages of the weights of masonry units currently produced. Heavyweight units will vary approximately 7 percent in weight from their average and lightweight units approximately 17 percent. In general, it appears that savings in dead weight of 30 to 40 percent may result through the use of masonry units of lightweight aggregate. The lighter masonry unit is also said to be conducive to lower construction costs deriving from greater ease of handling and higher output of mason labor. One estimate[11] of this effect on building costs places the increase in mason productivity at 20 percent over similar heavyweight units.

Thermal properties

While it is not the purpose of the paper to treat with insulating concretes as such, the heat insulating value that is obtained from the use of lightweight aggregate concrete in load-bearing masonry units is considered in some types of building construction. The published values of thermal conductivity, k, for oven-dry concretes in the unit weight range usually found in concrete masonry units indicate that those of lightweight aggregate are from 50 to 80 percent lower than those of sand and gravel. It should be borne in mind, however, that these advantages in k may not obtain in service because of the

TABLE 5—COMPARISON OF *U* COEFFICIENTS FOR VARIOUS WALLS*

Wall construction	Interior finish					
		½ in. plaster on				
	None	Wall	Gypsum board lath furred	Metal lath furred	½ in. rigid insulation furred	1 in. rigid insulation furred
Hollow concrete–8 in.						
Gravel aggregate	0.56	0.52	0.32	0.34	0.23	0.17
Cinder aggregate	0.41	0.39	0.27	0.28	0.20	0.15
Expanded slag, shale or clay	0.36	0.34	0.24	0.26	0.19	0.15
Frame						
2 x 4-in. studs 1-in. sheathing, wood siding	—	—	0.25	0.26	0.19	0.15

**U* values from ASHVE Guide[10] are expressed in Btu per hr per sq ft per deg F temperature difference between the air on the two sides and allow for an outside wind velocity of 15 mph.

effect of residual moisture at equilibrium with the service relative humidity. The reasons for determining conductivity, *k*, on the oven-dry basis are discussed in a later paper on insulating concretes.* Nevertheless, any marked differences in conductivity that might be indicated for concretes going into hollow block are not exhibited in the same proportion in the over-all heat transmission coefficient, *U*, for the built-up wall. In the hollow unit masonry wall the heat resistances of the core spaces and surface films contribute substantially to the over-all heat resistance. Thus, wide differences in the *k* value of the concrete tend to be obscured. As insulation and air spaces are added to the wall section, the differences become smaller. This is true of any type of wall construction—brick, stone, or frame. Table 5 gives the *U* coefficients for 8-in. hollow concrete block walls and illustrates the effect of added insulation. Considering the plain 8-in. wall, it will be seen that lightweight aggregate units may effect a saving of about 35 percent over heavyweight units. On the other hand, where the wall is furred and plastered on 1-in. rigid insulation the difference in insulating effect is about 12 percent. The tabulated values also indicate that the 8-in. hollow block wall of expanded slag, shale or clay aggregate concrete, plastered direct, is equal in heat resistance to the 8-in. wall of sand and gravel concrete plastered on furred metal lath. It appears, too, that when 8-in. hollow block walls of these lightweight aggregates are furred and plastered on metal lath, their heat resistance is comparable to the familiar wood frame wall, lathed and plastered.

Dry, granular or loose fill placed in the cores of hollow block masonry walls or in the space between the inner and outer elements of masonry cavity walls has been used with good insulating effect. Rowley and others[11] reported a reduction in the heat transmission coefficient, *U*, for a plain 8-in. hollow block masonry wall of expanded slag concrete from 0.35 to 0.22 Btu when the cores were filled with coarse expanded slag aggregate.

*Valore, R. C., Jr., "Insulating Concretes," ACI JOURNAL, Nov. 1956, *Proc.* V. 53, pp. 509-532.

Fire resistance

Considerable information is available on the fire resistance of concrete masonry wall construction and its load carrying capacity after severe exposure to fire, through published reports of the National Bureau of Standards, Underwriters' Laboratories, Inc., Portland Cement Association, and others. In general, this information shows substantial fire retarding properties for well constructed masonry walls composed of properly manufactured units of either heavyweight or lightweight aggregates.

Analyses of numerous fire test data on walls of various thicknesses and various unit designs indicate that fire resistance is dependent principally on the thickness of solid material in the heat path rather than the over-all thickness of the section comprised of solid material and hollow spaces. The National Building Code (1949) recommended by the National Board of Fire Underwriters estimates fire resistance ratings of walls and partitions of hollow concrete masonry on the basis of the "equivalent" thickness of solid material in the masonry unit. The "equivalent" thickness may be calculated by subtracting the volume of the core space of the unit from the total volume of the unit and dividing the result by the area of the face of the unit. Where walls are plastered, stuccoed, or faced with brick the thickness of these materials is included in determining the equivalent thickness. The relationship in Table 6, taken from the National Building Code (1949),[10] applies to plain hollow concrete masonry partitions and walls.

TABLE 6—ESTIMATED FIRE RESISTANCE RATINGS OF WALLS AND PARTITIONS OF HOLLOW CONCRETE MASONRY*

Aggregate	Minimum equivalent thickness in inches for ratings of			
	4 hr	3 hr	2 hr	1 hr
Expanded slag or pumice	4.7	4.0	3.2	2.1
Expanded clay or shale	5.7	4.8	3.8	2.6
Limestone, cinders or unexpanded slag	5.9	5.0	4.0	2.7
Calcareous gravel	6.2	5.3	4.2	2.8
Siliceous gravel	6.7	5.7	4.5	3.0

*From National Building Code (1949).

The *Standard for Concrete Masonry Units*, Underwriters' Laboratories, Inc.,[14] contains a somewhat similar comparison, Table 7. Here, fire resistance is related to the thicknesses of the face shells and webs of the masonry unit. The values given are minimum requirements for masonry units and aggregate types that have demonstrated, through standard fire tests,* eligibility for fire retardant classification.

It will be noted that in both comparisons (Tables 6 and 7) the use of lightweight aggregate concrete operates to reduce the thickness requirement for the various fire ratings. Fire tests have shown that the fire resistance of

*"Standard for Fire Tests of Building Construction and Materials of Underwriters' Laboratories, Inc.," Subject 263 (ASA No. A2-1).

TABLE 7—MINIMUM FACE SHELL AND WEB THICKNESSES OF HOLLOW CONCRETE MASONRY UNITS FOR VARIOUS FIRE RETARDANT CLASSIFICATIONS*

Type of aggregate	Class D-2 retardants		Class C-3 retardants		Class B-4 retardants†	
	Face shell minimum, in.	Web minimum, in.	Face shell minimum, in.	Web minimum, in.	Face shell minimum, in.	Web minimum, in.
Natural, by-product, and processed, except those listed below	1¼	1	1½	1	2⅛	1½
Burned clay or shale	1⅛	1	1⅜	1		
Expanded slag	1⅛	1	1⅜	1	1⅝	1
Pumice					1¼	1

*The minimum face shell for the largest core hole at the thinnest point. This may be the average of the two face shells in the same core. The end and internal webs in each core hole at the thinnest point.

†Class D-2 includes units classified as at least 2-hr, but less than 3-hr retardants. Class C-3 includes units classified as at least 3-hr, but less than 4-hr retardants. Class B-4 includes units classified as at least 4-hr, but less than 8-hr retardants.

hollow concrete masonry unit walls may be improved by filling the cores with dry lightweight aggregate. Underwriters' Laboratories, Inc., allows a 4-hr rating for Class D-2 or C-3 retardants when the cores are filled with dry, loose expanded slag.*

Menzel,[15] in discussing the results of his tests, points out that for concrete of a given cement content and aggregate grading, the temperature rise upon exposure is the resultant of several factors which vary with the type of aggregate used: conductivity, heat capacity, moisture, decarbonization, and fusion. High conductivity and high heat capacity (and vice versa) in a concrete will produce opposing effects on the temperature rise. Moisture has a strong retarding influence on temperature rise. Decarbonization applies to the calcium and magnesium carbonates in the aggregate of the concrete and since the calcining process is endothermic, it has a retarding action on the transmission of heat. Fusion of aggregate absorbs heat. However, this influence can largely be discounted since it rarely occurs below 2200 F or before 6 hr of a standard fire exposure.

In the case of lightweight aggregate concretes the advantage of lower conductivity will be opposed by the disadvantages of lower heat capacity. It appears, however, that in most of the lightweight aggregate concretes the influence of lower conductivity on heat transfer predominates. It appears, too, that this influence might be reinforced by the greater capacity of lightweight aggregates for the storage of moisture. Menzel found that porous, cellular aggregate materials retain more water than dense materials even after long drying periods at the temperature and humidity of a heated building.

Other characteristics of lightweight aggregate concrete which have sig-

*UL Guide No. 40-U-18.

nificance in consideration of the fire resistance of concrete masonry walls and partitions of the nonbearing type are the lower coefficients of thermal expansion and lower elastic moduli. Where walls are constructed tightly between rigid columns and floor beams of a building, such walls may become heavily restrained at four edges during fire exposure. This restraint in opposition to high thermal expansion could operate to produce crushing loads. Menzel concluded that nonbearing walls of concrete masonry units made with aggregates of relatively low thermal expansion characteristics, such as cinders, expanded slag, and expanded clay or shale, could be expected to serve as effective barriers to fire and to resist the restraining loads which may be reasonably expected under fire exposure conditions.

Acoustic properties

The sound absorbing characteristics of building materials are of interest in the acoustical correction of auditoriums and in the control of air-borne noise in office rooms or manufacturing areas. Concrete masonry wall construction composed of units of porous surface texture is being effectively utilized for these purposes. Porosity has been shown as the chief property affecting the sound absorbing efficiency of materials.[8,9] Sound absorption takes place during sound wave reflection. The property of porosity functions in this instance by converting the energy of sound waves penetrating the surface of the material to heat energy through the frictional resistance that is set up by minute channels. Larger and continuous pores which permit sound energy to become "lost" by passage through the section also contribute to sound absorption. Tests have indicated that it is possible to obtain a high degree of sound absorbing efficiency in masonry unit concretes with heavy aggregates by special control of the grading, the concrete mix, and the compaction, but it is evidently impractical to direct the manufacture of masonry units of heavy aggregates specifically to high acoustical efficiency.

Lightweight aggregates apparently provide natural assistance in the problem. Published data on the sound absorption of masonry walls of commercial grade units indicate that those of lightweight aggregates have higher sound absorption efficiency than those of heavyweight aggregates. The average* coefficient for unpainted walls of commercial grade lightweight concrete masonry units ranges from about 0.45 to 0.60 with 0.50 a fair representation. Walls of heavyweight aggregate units that are porous by virtue of special aggregate grading, the amount of water in the mix, and concrete compaction may exhibit equal or higher sound absorption efficiency, but the major proportion of heavyweight aggregate units produced are such that the average* sound absorption coefficient will fall in the range from about 0.20 to 0.35.

Paint coatings that tend to close the pores and interstices of the masonry surface will reduce the sound absorbing efficiency. If the coating is applied in a manner (spray or roller applicator painting) such that an appreciable number of surface openings remain, sound absorption efficiency will not be reduced by more than 15 percent. The sound absorption of coarse textured

*The average of coefficients determined at 250, 500, 1000, 2000, and 4000 cycles per sec sound frequency.

masonry units is depreciated to a lesser degree by paint treatment than fine textured units because the paint does not readily bridge the larger surface openings. If the material behind the surface is essentially porous, relatively few surface openings are needed to realize a substantial proportion of the original absorption efficiency. The "metal pan" acoustical assembly, which is composed of a perforated metal facing with a backing of porous mineral wool, makes use of this principle. Tests of this system have established that it is possible to cover a substantial proportion of the surface of sound absorbent material with a perforated rigid membrane with no significant loss in efficiency.[16]

TABLE 8—COEFFICIENTS OF THERMAL EXPANSION OF CONCRETES FOR MASONRY UNITS CONTAINING VARIOUS AGGREGATES[3]

Aggregate	Coefficient of thermal expansion*
Cinders	2.1×10^{-6}
Expanded shale	3.9×10^{-6}
Expanded slag	3.9×10^{-6}
Sand and gravel	5.5×10^{-6}
Crushed limestone	5.1×10^{-6}

*Determined over a temperature range of from −7 F to 125 F.

Sound insulating properties

High acoustical value in concrete masonry wall construction which is attained by virtue of pores or interstices that are continuous through the concrete section may operate to provide lower resistance to sound transmission. The property of sound insulation is discussed in a later paper on insulating concrete.*

Volume change

Interest in the volume change properties of concrete masonry units stems principally from the problem of shrinkage cracking in masonry walls. Shrinkage cracks may result from stresses induced either by restrained thermal shrinkage or restrained drying shrinkage or by a combination of both. Cracking occurs when stresses produced by these effects exceed the strength of the stressed section.

The relative importance of either factor, thermal or drying shrinkage, in contributing to the problem varies considerably with the service environment and the intrinsic properties of the concrete. In northern regions of the country where the extreme range of temperature in wall construction may exceed 100 F and where sudden temperature drops of as much as 50 F are possible, the thermal volume change properties of the concrete may dominate. On the other hand, in applications where the environment is characterized by fairly uniform temperature and low relative humidity drying shrinkage may be more important.

With respect to thermal volume stability, lightweight aggregate concretes for masonry units, as a class, possess some advantage over their heavyweight aggregate counterparts (Table 8). This advantage is opposed by their generally higher drying shrinkage. Some lightweight types compare quite closely

*Valore, R. C., Jr., "Insulating Concretes," ACI JOURNAL, Nov. 1956, *Proc.* V. 53, pp. 509-532.

with the heavyweight group in drying shrinkage while others have exceedingly high shrinkage. Copeland[17] estimates that a fair representation of heavyweight commercial block, steam cured at atmospheric pressure, will shrink approximately 330 millionths in. per in. from the saturated condition to moisture-volume equilibrium with 50 percent relative humidity. A similar sampling of lightweight aggregate types under similar conditions of initial moisture content and drying will shrink about 450 millionths. Some data developed at the Research Foundation of the University of Toledo[19] suggest that the higher drying shrinkage of walls composed of lightweight units is compensated, in part, by greater extensibility. The modulus of elasticity of lightweight aggregate concrete for masonry units ranges from ½ to ⅔ that of heavyweight aggregate concrete.

It is possible to alleviate to a considerable degree the shrinkage cracking in masonry walls due to drying by reducing the moisture content of the masonry units before they are laid. The ASTM and federal specifications for concrete masonry units limit the moisture content of delivered block to 40 percent of the total absorption for this specific purpose (Table 2). As previously mentioned, some requirements[20] have established the moisture limit at 30 percent of total absorption. In instances where the service environment is characterized by low humidity, such as in buildings heated in winter without humidification, moisture levels as low as 20 percent of the total absorption may be required. Low moisture content levels in delivered block may present some problems for the contractor in preserving these levels under some construction-site moisture exposures.

The use of bond beams, horizontal joint reinforcement, and control joints as added measures against the development of cracks in concrete masonry walls induced by volume change is gaining in scope. Obviously, such measures add to the cost of masonry wall construction. But in long runs of walling, particularly those having a plurality of openings, the value of these measures for controlling cracking has been demonstrated. In areas of the country where hurricanes, severe storms or earthquakes are not unusual, the use of bond beams and joint reinforcing and other reinforcing measures are required by building code. The proper construction of bond beams and control joints for concrete masonry walls is discussed in references 10 and 20.

Special applications

It is not within the province of this paper to discuss reinforced structural lightweight aggregate concrete, but it appears appropriate to point out that large numbers of lightweight units are being used for prefabricated structural floor, roof, and wall elements. Lower haulage and erection costs resulting from the reduced weight appear as the principal factors in these applications, although the heat insulating and acoustic properties and fire resistance are said to contribute. Special filler block and soffit tile of lightweight concrete are also used in the *in situ* construction of reinforced concrete floors and roofs. The units are shaped or may be arranged in the forms to provide the

ribbed slab type of floor or roof design. The resulting flat ceiling surface may be plastered, or when untreated or lightly painted has usable sound absorption value. Such floors and roofs are said to permit higher fire ratings compared to regular cast-in-place ribbed concrete slab construction.

SUMMARY

In summarizing this discussion the following statements seem justified:

1. The importance and value of lightweight aggregates in the production and use of concrete masonry units is well established. Of the estimated 1954 production of over 1¾ billion 8 x 8 x 16-in. masonry unit equivalents, about 50 percent were of lightweight aggregate concrete.

2. Due to differences between the several types of lightweight aggregates with respect to particle surface texture and shape, proper grading of lightweight aggregate for a given application can only be generalized. Specific grading requirements are better determined through analysis and experimentation.

3. Lightweight aggregates for use in the manufacture of concrete masonry units require somewhat more attention in the control of segregation and in the preparation of concrete (prewetting) than heavyweight aggregate.

4. The literature indicates that lightweight aggregates enhance thermal resistance, sound absorption, and fire retardant properties of concrete masonry walls.

5. In general, concrete masonry units composed of lightweight aggregates exhibit greater drying shrinkage than those of heavyweight aggregates. Some compensation for this property relative to drying shrinkage cracking of masonry walls obtains from the lower moduli of elasticity and greater extensibility of lightweight aggregate concretes.

6. The incidence of cracking in concrete masonry walls arising from drying shrinkage can be minimized by adequate predrying of masonry units. Current specifications limit the moisture content of delivered masonry units for this purpose.

REFERENCES

1. Davis, R. E., and Kelly, J. W., "Lightweight Aggregates," Symposium on Mineral Aggregates, *Special Technical Publication* No. 83, ASTM, 1948, p. 160.

2. "Lightweight Aggregate Concrete," Housing and Home Finance Agency, Washington, D. C., Aug. 1949.

3. Wendt, Kurt F., and Woodworth, Paul M., "Tests on Concrete Masonry Units Using Tamping and Vibration Molding Methods," ACI JOURNAL, Nov. 1939, *Proc.* V. 36, pp. 121-163.

4. *Technical Bulletin* No. 5, National Concrete Masonry Association, Chicago, Aug. 24, 1955.

5. Ehle, J. C., "Developments in the Manufacture and Technology of Concrete Masonry Units," ACI JOURNAL, Apr. 1949, *Proc.* V. 45, pp. 613-620.

6. Menzel, Carl A., "Studies of High Pressure Steam Curing of Tamped Hollow Concrete Block," ACI JOURNAL, Sept.-Oct. 1935, *Proc.* V. 32, pp. 51-64.

7. *Technical Report* No. 40, National Concrete Masonry Association, Chicago, Jan. 19, 1953.

8. Watson, F. R., and Morrical, K. C., "Sound Absorbing Value of Portland Cement Concrete," ACI JOURNAL, May-June 1936, *Proc.* V. 32, pp. 659-671.

9. Watson, F. R., *Acoustics of Buildings*, John Wiley & Sons, Inc., New York, 1923.

10. *Concrete Masonry Handbook*, AIA File No. 10-C, Portland Cement Association, Chicago, 1951.

11. Whitaker, T., "Lightweight Concrete In America," *National Building Studies Special Report* No. 13, Department of Scientific and Industrial Research, London, England, 1953.

12. *American Standard Building Code Requirements for Masonry*, ASA A41.1—1953.

13. *American Society of Heating and Ventilating Engineers Guide*, 1953 Edition, pp. 172 and 188.

14. *Standard for Concrete Masonry Units*, Underwriters' Laboratories, Inc., National Board of Fire Underwriters, Subject 618, 4th Edition, Dec. 1953.

15. Menzel, Carl A., "Tests of the Fire Resistance and Strength of Walls of Concrete Masonry Units," Portland Cement Association, Chicago, 1934.

16. Sabine, Hale J., "Manufacture and Distribution of Acoustical Materials Over the Past 25 Years," *Journal of the Acoustical Society of America*, V. 26, No. 5, Sept. 1954, pp. 657-661.

17. Copeland, R. E., "The Problem of Shrinkage Cracking," *Technical Report* No. 20A, National Concrete Masonry Association, Chicago, 1951.

18. "Relation of Shrinkage to Moisture Content Concrete Masonry Units," *Housing Research Paper* No. 24, Housing and Home Finance Agency, Washington, D. C., Mar. 1953.

19. "Shrinkage Characteristics of Concrete Masonry Walls," *Housing Research Paper* No. 34, Housing and Home Finance Agency, Washington, D. C., Apr. 1954.

20. "Requirements for Concrete Masonry Construction," *National Bureau of Standards Report* No. 3079, Jan., 1954.

21. Seaton, S. G., "Study of Causes and Prevention of Staining and Pop-Outs in Cinder Concrete," ACI JOURNAL, Jan. 1948, *Proc.* V. 44, pp. 361-380.

For such discussion of this paper as may develop please see June 1957 JOURNAL.

Title No. 53-27

The final paper in a series of four on lightweight concrete

Insulating Concretes*

By R. C. VALORE, JR.†

SYNOPSIS

Mix proportioning, mixing methods, and physical properties are reviewed for insulating concretes of the types in which structural properties are secondary to thermal insulation value. Densities for four compressive strength ranges are given. Thermal conductivity data from different sources are compared as a function of density. Data are reviewed on other strength properties —elasticity, water absorption, frost resistance, drying shrinkage, and thermal expansion.

Properties often considered in relation to insulating concretes are fire resistance, sound insulation, and sound absorption. Data are given on these properties for lightweight aggregate concretes generally and for concretes used primarily for thermal insulation. The sections on mechanics of sound insulation and absorption are intended to dispel some misconceptions on these subjects in relation to other physical properties of concrete.

INTRODUCTION

The term "insulation" is used variously, to denote retardation of the transfer of heat, fire, sound, or other physical phenomena. It is usually in the thermal sense that the term applies in "insulating" concretes, designed to function primarily as insulative, rather than structural, materials. Fire resistance of concrete depends in part on resistance to heat flow at elevated temperatures, but may also depend on the relative thermochemical and thermophysical stability of the material as well.

Inclusion of sound insulation in the scope of this paper implies an association of that property with lightweight materials that is common, but erroneous. Comprehensive study of the literature reveals no material that can rightly be called "sound-insulating concrete," lightweight or otherwise. Confusion of the distinctly different acoustic properties, sound absorption and sound insulation, may account for the misconception.

Lightweight concretes intended for use as thermal insulation may range in 28-day compressive strength from less than 100 to 500, or at most, 1000 psi, and in density (oven-dry) from 15 to 90 lb per cu ft.

In addition to thermal conductivity, fire resistance and acoustic properties of lightweight concretes are described in moderate detail. Radiation shielding

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is not within the scope of this study; this property of concrete was reviewed in a recent paper.[1]

Insulating concretes may be considered, according to composition, in three groups:

Group I—Concrete weighing (oven-dry) 15 to 50 lb per cu ft, made with expanded perlite or vermiculite aggregate.

Group II—Concretes weighing (oven-dry) 45 to 90 lb per cu ft, made with aggregates prepared by calcining, sintering, or otherwise expanding such products as blast-furnace slag, clay, diatomite, fly ash, shale or slate; or made with aggregates processed from such natural materials as pumice, scoria, or tuff.

Group III—Concretes weighing (oven-dry) 15 to 90 lb per cu ft, prepared by imparting to neat cement paste or cement-sand mortar a uniformly cellular structure by incorporating air, using mechanical means, or other gas cells, using chemical methods, in the mixture during or immediately following mixing.*

Groups I and II correspond to the classification of aggregates contained in ASTM C 332-54T on lightweight aggregates for insulating concrete.[3] Group III concretes are moist-cured "foam" and "gas" cellular concretes.

COMPOSITION AND PREPARATION OF INSULATING CONCRETES

Aggregates

Washa[4] summarizes the processing and the ASTM grading and unit weight requirements of aggregates for insulating concretes. The characteristics of these aggregates have also been described by Price and Cordon,[5] Kluge, Sparks, and Tuma,[6] and the Housing and Home Finance Agency.[7]†

The ASTM grading requirements have been modified to a considerable degree for some experimental insulating concretes.[8] Relatively large percentages of entrained air (20 to 35 percent) lowered drastically or eliminated entirely the requirement for fine aggregate in insulating concretes made with Group II aggregates. Similarly, the minus No. 50-sieve fractions of perlite or vermiculite aggregate may be omitted in mixes designed to have densities (oven-dry) of 25 lb per cu ft or less.

Cellular concretes either contain no aggregate (exclusive of air) or are made with dense aggregate passing the No. 4 sieve; finer sands also have been used. Certain pozzolans may also be used advantageously in moist-cured cellular concretes. Extremely high air contents obviate grading limitations other than that of maximum particle size. The maximum size of aggregate particles for satisfactory preparation of "gas" (chemically formed) cellular concretes is usually appreciably smaller than the No. 4 sieve opening ($\frac{3}{16}$ in.) because of the tendency for coarser particles to settle in the thin-textured slurry before completion of the gas-forming process.

Some of the general requirements for aggregates[9] used in structural lightweight concretes may be waived in certain applications of insulating concretes, which are often neither exposed to weather nor required to support appreciable loads.

*Cellular concretes considered here are moist-cured as differentiated from high-pressure steam-cured cellular materials. The latter are described in a recent paper.[2]

†The HHFA report is a comprehensive summary of references 5 and 6.

Additional characteristics, the requirement for which is considered debatable among some aggregate producers and users, are "hardness" and "yield." One measure of hardness is the ability of an aggregate to resist crushing due to compressive loading when laterally restrained.[4] Brouk[10] recommends 75 psi minimum crushing strength for 1 in. compaction for Group I aggregates to prevent excessive breakdown of aggregate in transit or during mixing. Breakdown of aggregate can cause reduced strength in concrete of a given specific weight by increasing the effective density of the aggregate and by increasing the proportion of fine material, thereby increasing both the air and water requirements. Instances of serious breakdown of perlite aggregate while passing through a screw-conveyor have been reported.[11] Reduction of strength by as much as 50 percent has been attributed to breakdown of a Group I aggregate during prolonged mixing.

"Yield" is the relationship of the volume of concrete to dry, loose volume of aggregate: if 1 cu ft of concrete contains 1 cu ft of aggregate, the yield is 100 percent. Breakdown of aggregate lowers yield, but increasing the quantity of entrained air increases yield. Group I concretes are often designed for 100 percent yield, but yields of 80 to 120 percent may also be recommended. Yields as high as 230 percent have been obtained for perlite concrete weighing (oven-dry) 22 lb per cu ft and made with preformed foam;[8] cellular neat cement paste might be considered to be an example of infinite yield.

Combinations of Group I and Group II aggregates may be used in insulating concretes. Perlite and expanded shales, or sand and Group I or Group II aggregates have been used in the preparation of concrete in the laboratory. A potential advantage of using aggregate blends is the preparation of concretes having specific weights in a range not easily or economically obtained using a single type of aggregate. Nail-holding ability has also been advanced as a possible advantage of aggregate-blend concretes. Certain compromises in strength, drying shrinkage, and cost may result from the use of aggregate blends.

Preparation of concretes; mixing procedures; air entrainment

Perlite and vermiculite concretes are mixed in either a horizontal-drum rotating-paddle type of mortar or plaster mixer, which is usually preferred, or a tilt-drum, fixed-blade concrete mixer.

Water, air-entraining agent, and cement, in order, are placed in the mixer with blades turning, and the dry aggregate is then added and the concrete mixed until it is homogeneous.*

In the rotating-paddle mixer, increasing the speed of rotation (within limits) provides more efficient air entrainment and reduces the quantity of air-entraining agent required. The percentage of air entrained may also depend on the size of mixer and duration of mixing. The proper quantity of air-entraining agent may have to be determined by trial and error for the particular mixer used and the density desired.

*The air-entraining agent is added as an aqueous solution at the mixer when perlite is used, but vermiculite aggregate usually is treated to contain the "correct proportion" of air-entraining agent by the aggregate producer.

Another method of air entrainment is the replacement of conventional air-entraining agents with preformed "air foam," as employed with some cellular concretes.[2,12-14] This method has been tried in the field and laboratory in making perlite roof-fill concretes with encouraging results. Foam was added with water, or following water and cement; dry aggregate was added last, as usual. No other alteration of field practice was required. Indicated advantages of this method were a close control of density and stability of concrete volume prior to hardening.

Group II aggregates may be used dry but control of mix properties is easier if the aggregates are moistened before mixing. Moistening periods as short as 15 to 30 min. minimize decrease in volume of concrete before hardening.[8] These aggregates generally require the use of a tilt-drum concrete mixer because of the presence of coarse particles. Control of air content may be difficult and may depend on the amount of water in the mixture (exclusive of water absorbed by the aggregate). It may be desirable to begin mixing with less water than is deemed necessary and to add the remaining water, as mixing proceeds, in amounts necessary only to provide the proper degree of air entrainment and workability, as estimated visually or by trial weighings. These precautions may be necessary even if the amount of surface moisture introduced with the aggregate is fairly constant from batch to batch.

With Group II aggregates the quantity of air-entraining agent required depends upon the type and gradation of aggregate. The presence of fine aggregate imposes a "ceiling" on the quantity of entrained air, but control of air content in some "no-fines" mixtures has been difficult because of the ease with which extremely high proportions of air may be entrained.[8] The percentage of air entrained may also vary with size of mixer, size of batch, and duration of mixing.

Allowance should be made for a decrease in air content during the time between the end of the mixing period and placement of the concrete in the forms. This loss may vary with the type of air-entraining agent used.

The mixing of cellular concretes has been described in detail in a previous paper.[2] Several types of mixing are employed: (a) high-speed mixing during which air entrainment is effected by the beating action of mixer paddles upon the ingredients in the presence of a foaming (air-entraining) agent; (b) moderate to high-speed mixing or blending, in which a stable air-foam is the mix ingredient that provides the cellular structure; and (c) moderate to high-speed mixing of mixtures containing such gas-forming chemicals as aluminum powder or hydrogen perixode and calcium hypochlorite. When a gas-forming chemical is used the mixture increases in volume while in the forms. For field applications the so-called "foam" concretes, either "mix-foamed" or "prefoamed" may be more convenient to use and their densities more easily controlled than is the case with "gas" concretes.

Properties of freshly mixed insulating concretes

Proportions of insulating concrete are generally given as the ratio of bags of cement to dry loose volume of aggregate (cu ft). Perlite and vermiculite

aggregates are usually delivered in 4-cu ft bags. Mixes having proportions ranging from 1:4 to 1:8 for perlite[15] and 1:3 to 1:16 for vermiculite[16] have been described. However, the number of mix designs generally used is relatively small. Brouk[10] describes 1:4 and 1:6 perlite mixes, and recommends the 1:6 mix for roof-fill insulation applications. The 1:5, 1:6, and 1:8 mixes are now often used for vermiculite roof fill.[17] The higher proportions of aggregate are usually recommended for lower-density concrete. The use of preformed foam, however, permits the use of relatively low proportions of aggregate in concretes of low density; *e.g.*, 1:2.5 for Group I concretes weighing (oven-dry) 20 to 30 lb per cu ft. Group I concretes having dry densities of 20 to 30 lb per cu ft and proportions of 1:5 or 1:6 have cement factors of 3 to 5 bags per cu yd of concrete; for given proportions and density the cement factor will vary with the properties of the aggregate within each mineral type, as well as with the air content.

Although not included in the usually specified proportions, because its precise measurement is difficult, the air content of Group I concretes may constitute an appreciable proportion of the volume of a mixture. Recent tests have indicated that air contents of 25 to 30 percent are required in some perlite mixtures to attain specified densities for roof fills.[8]

The proportions of Group II concretes are also based on volumes of dry loose materials, even when the aggregate is batched in a moist condition. The relation of the dry to the "as-used" moist unit weight of aggregate should be known to maintain control of proportions.* The range in satisfactory proportions when using Group II aggregates can vary considerably. Proportions of 1:4 to 1:14 have been used for insulating concretes made with pumice, expanded slag, and expanded shale aggregates.[8,11,18-21] "No-fines" mixtures with 20 to 35 percent of entrained air have had proportions of 1:4 or 1:5. The cement factors of Group II insulating concretes have ranged from 2 to 6 bags per cu yd, depending upon the air content, aggregate gradation, and proportions.

The relationships between proportions, density, and cement factor of cellular concretes have been published recently.[2] Proportions of cement to aggregate ranging from 1:0 (neat) to 1:4 (by weight) have been used. Neat cement cellular concrete contains almost 5 bags of cement per cu yd of concrete at a density (oven dry) of 20 lb per cu ft and about 10 bags per cu yd at 40 lb per cu ft.

Cement-sand cellular concretes having proportions of 1:2, 1:3, and 1:4 and dry densities of 55, 75, and 90 lb per cu ft, respectively, have an approximate cement factor of 5 bags per cu yd.

The water requirements for insulating concretes will depend upon the type and grading of aggregate and upon the degree of air entrainment.

Among concretes made with Group I aggregates the water requirement includes that absorbed by the dry aggregate; it is difficult, if not impossible,

*Proportions by dry, loose volumes will be different for a given mix, depending upon whether the fine and coarse aggregates are considered separately or combined.

to arrive at an accurate value for net water-cement ratio. For perlite roof-fill concrete (1:6 mix) Brouk[10] has given water requirements ranging from 47 to 95 gal. per cu yd of concrete, the highest quantities being required for the softest aggregates and those containing the highest proportions of fine particles. Perlite mixes described by Kluge, Sparks, and Tuma[6] required 73 gal. of water per cu yd of concrete (proportions 1:2.9 to 1:5.5) and 92 gal. per cu yd (1:10.3). The range of water requirements given by Price and Cordon[5] was from 40 to over 100 gal. per cu yd for concretes of various proportions made with a number of different perlites.

The Perlite Institute[15] recommends 54 to 61 gal. of water per cu yd of concrete for proportions ranging from 1:4 to 1:8.

Water requirements per cubic yard of vermiculite concrete mixes prepared for the Vermiculite Institute[16] did not vary greatly with proportions. For proportions 1:3 to 1:16 the range of water requirements was about 75 to 105 gal. per cu yd of concrete; each of these mixes ranged from $\frac{1}{8}$ in. to about 9 in. in slump. Requirements of water per unit of volume for 1:4 to 1:10 vermiculite concrete mixes were generally lower for material of 6-in. slump than for higher or lower slumps. This fact was probably related to the efficiency of air entrainment as a function of consistency of the concrete. The water required per bag of cement increased as the proportion of vermiculite increased. For given proportions, however, the additional water required in a batch to bring about an increase in slump from 3 to 6 in. resulted in a lowered water content per cubic yard of concrete apparently because of more efficient entrainment of air and increased yield. The yield increased as the slump increased for this whole series of mixtures. The recommended water requirement for "Zonolite" vermiculite concrete (for 100 percent yield) is 95 to 100 gal. per cu yd ($\pm$ 20 percent).[17]

Water contents of 100 to 110 gal. per cu yd were reported by Kluge, Sparks, and Tuma,[6] and 60 to 75 gal. per cu yd by Price and Cordon[5] for vermiculite concretes. The disparity was probably due to the use of smaller amounts of air-entraining agent and less entrained air in the mixes of Kluge, Sparks, and Tuma, with the necessity for using greater amounts of water to attain satisfactory workability.

The water requirements of Group II concretes depend, as for Group I, upon the proportions of entrained air, and, additionally, upon the moisture condition of the aggregate prior to mixing. Water contents (*not* including water absorbed by aggregates) of no-fines concretes made with pumice, expanded slag, and expanded shale, and containing 5 bags of cement per cu yd and 20 to 35 percent of entrained air, have been 30 to 35 gal. per cu yd.[8] Total water (including that absorbed by aggregates) ranged from about 40 gal. per cu yd for expanded shale aggregates to about 60 gal. per cu yd for pumice aggregates. Water contents (not including that absorbed by aggregates) as low as 27 gal. per cu yd have been reported for other mixtures containing both fine and coarse Group II aggregates.[18-21]

The water requirements for cellular concretes depend upon the particle size of the aggregate used, the proportions, and density of the concrete.

Water requirements for cellular neat cement range from 40 to 60 gal. per cu yd for dry densities of 20 to 45 lb per cu ft, respectively.[14] For 1:2 and 1:4 cellular cement-sand mixtures, total water requirements were 25 to 45 gal. per cu yd, the amount required varying with density of concrete in the range 50 to 100 lb per cu ft.

The importance of the water requirement must be judged according to each application of insulating concrete. If a limited period is available for drying the concrete before applying such waterproofing as built-up asphalt-felt roofing, large quantities of evaporable water remaining in the concrete may seriously lower the insulating efficiency and possibly cause later damage to the waterproofing membrane.

The workability of insulating concretes containing relatively large amounts of entrained air is generally excellent, even for no-fines mixtures made with Group II aggregates. Slumps of 5 to 7 in. may be quite satisfactory; the mixtures are usually highly plastic and homogeneous and of a somewhat "spattering" type of consistency. Appearance of the mixture may be a more reliable indication of consistency than a slump test. Bleeding and segregation problems will not ordinarily be present because of high air contents. These mixtures will usually be readily and easily placed simply by "pouring" and screeding, without further consolidation.

Cellular concretes are handled virtually as liquids and are simply poured into place without further consolidation. The application of these concretes may be limited to fairly flat and reasonably watertight supporting elements.

The moist curing of insulating concretes rarely exceeds 7 days, and frequently no moist curing is provided. In roof fill and other applications where it is desirable for the concrete to be as dry as is conveniently possible before the application of waterproofing materials, uncertainties of weather require that elaborate curing be omitted.

Precast insulating concretes may be subjected to high-temperature curing. The use of autoclave curing for cellular concretes has become widespread in Europe.[2]

PHYSICAL PROPERTIES OF INSULATING CONCRETES

Density

A practice employed in Europe and in this country is to define the dry density of cellular and some other lightweight concretes as that obtained by drying in an oven at about 220 F until the loss in weight during a 24-hr period is less than 1 percent. Oven-drying at 212 to 248 F, beginning when the concrete is 28 days old is specified in ASTM C 332-54T, for specimens used for density and thermal conductivity determinations.[3]

Table 1 contains average values for the density of freshly-mixed and oven-dry concretes made with Group I and Group II aggregates, and for moist-cured cellular concretes.

TABLE 1—AVERAGE PROPORTIONS AND DENSITY OF INSULATING CONCRETES

Aggregate	Proportions	Cement factors, bags per cu yd	Density, lb per cu ft	
			Fresh	Oven-dry
Perlite*	1:4	6.7	50	35
Perlite*	1:5	5.3	45	30
Perlite*	1:6	4.5	40	27
Perlite*	1:8	3.4	36	22
Vermiculite*	1:4	6.7	60	38
Vermiculite*	1:6	5.1	51	28
Vermiculite*	1:8	3.4	48	23
None (cellular, neat)	1:0	4.8	28	20
None (cellular, neat)	1:0	7.3	40	30
Pumice	1:5–1:9	5.0–3.3	80–75	63–57
Pumice (aerated, no-fines)	1:5	5.5	62	50
Expanded slag	1:5–1:9	5.0–3.0	85–70	75–60
Expanded slag (aerated no-fines)	1:5	5.0	64	57
Expanded shale	1:3–1:14	5.4–2.0	83–60	72–54
Expanded shale (aerated, no-fines)	1:5	5.0	72	65

*For mixes of 100 percent yield; these aggregates are often used in mixes having yields as much as 15 percent higher or lower than 100 percent, in which case the cement factor and density would vary inversely with the yield.

Although the oven-dry condition may not readily be attained in a structure, it is a reproducible index of merit for control purposes. Table 1 indicates that controlling the oven-dry density of the freshly-mixed concrete must be based on a knowledge of the probable loss in weight on drying of the mixture used. The greatest loss in weight is shown by the Group I concretes. A recent paper by Copeland and Bragg[22] on the nonevaporable water content of concrete as a function of age offers means for estimating the oven-dry weight if the cement and total water contents are known.

Strength

Although strength is often of secondary importance in insulating concretes, it is usually given prominent treatment in literature on these materials. The effects of type of cement, type of curing, specimen size, and moisture condition, considered in earlier studies of Valore,[2] Tucker,[23] and Bessey and Dilnot,[24] are considered applicable to the present materials.

It is current practice to test some insulating concretes after they have been oven-dried.[25] This practice differs from that for other concretes. There appears to be no valid reason why insulating concretes should not be tested at 28 days (7 days if high-early-strength cement is used) after curing in air at ordinary temperature. The specimens may be rectangular or cylindrical prisms with slenderness ratios between 1.5 and 2.[23] Specimens may be easily prepared for testing by grinding bearing surfaces plane and smooth. A yielding material such as leather belting or fiberboard may be interposed between bearing surfaces and testing machine. In this way possible adverse effects of heat or moisture, due to the use of other capping materials, may be avoided.

Strength requirements for insulating concretes depend on the installation. Insulation for underground steam lines needs little strength; minimum strengths of 80 psi are recommended in some cases but even lower values may

be satisfactory. Roof fill over structural slabs requires sufficient early strength to assure that traffic of workmen applying waterproofing materials will not crush the concrete. Compressive strengths specified for roof fills may range from 100 to 500 psi, but in many applications 100 to 200 psi will be adequate. The U. S. Army Corps of Engineers requirement for insulating concrete to be used in building construction is 125 psi compressive strength and 60 lb per cu ft maximum density when dried at 200 F to constant weight.[25]

There is no simple relationship among proportions, cement factor, density, and strength for insulating concretes. For Group I concretes the lowest ratios of cement to aggregate are usually recommended for concretes of lowest densities and strengths. However, because the relation of cost of Group I aggregates to cost of cement is different from that for Group II and denser aggregates, consideration may be given to the use of higher cement factors and lower aggregate contents for low density Group I concretes. Perlite and vermiculite mixes proportioned 1:2.5 have been prepared and have weighed 20 to 30 lb per cu ft, oven dry; preformed foam was used for air entrainment.[8] This type of mixture is a compromise in cost between the currently recommended 1:6 perlite mix for this density range and less costly cellular neat cement concretes.

For Group II concretes also, a given strength and density may be obtained with a wide range of cement factors and proportions, but the cost will usually vary directly with cement content.

Average oven-dry densities of insulating concretes in four ranges of 28-day compressive strength are presented in Table 2; if high-early-strength cement is used the strength ranges shown are for 7 days.

Group II aggregates may vary appreciably in specific gravity and unit weight, depending upon aggregate source, process of manufacture, and gradation of aggregate. It may be desirable to select Group II aggregates for insulating concretes from among those having unit weights lower than average for their respective mineral classifications.

A strength property thought by some to be more nearly indicative of the ability of insulating concretes to withstand foot traffic is the "indentation strength" as described recently by Brouk.[10] The indentation strength is the load required to cause "failure" (evidence of crushing) when applied through a flat, circular disc surface of one square inch. The U. S. Army Corps of

TABLE 2—APPROXIMATE OVEN-DRY DENSITIES FOR INSULATING CONCRETES IN FOUR RANGES OF COMPRESSIVE STRENGTH*

Aggregate and proportions	75 to 125 psi	125 to 250 psi	250 to 500 psi	500 to 1000 psi
Perlite 1:3 to 1:10	20 to 25	22 to 30	27 to 40	35 to 60
Vermiculite† 1:3 to 1:10	22 to 28	24 to 32	29 to 45	37 to 60
Sand (cellular) 1:4	50 to 60	55 to 70	60 to 80	70 to 95
None (cellular)† 1:0	20 to 25	22 to 30	28 to 45	37 to 60
Pumice 1:5	40 to 45	40 to 50	45 to 60	45 to 70
Expanded slag 1:5 to 1:9	55 to 65	60 to 70	60 to 75	65 to 80
Expanded shale 1:5 to 1:14	55 to 60	55 to 65	55 to 70	60 to 75

*For 6 x 12-in. cylinders tested in air-dry condition; for normal portland cement (Type I), 28-day strengths are given and for high-early-strength cement (Type III) 7-day strengths are given.
†Not usually designed for strength exceeding 750 psi.

Engineers requirement for indentation strength of insulating concrete used for building construction is 140 psi when the concrete is "cured for 28 days and dried to constant weight."[25] Data for perlite[10] and vermiculite[16] concretes indicate that the indentation strength is usually at least 50 percent higher than the corresponding compressive strength.

Flexural strength and bond strength data for insulating concretes appear meager.

The available data on modulus of rupture appear to be quite erratic, especially for concretes made with Group I aggregates, and pumice, and for moist-cured cellular neat cement. Large variation in flexural strength may be due to transverse shrinkage cracks in specimens of these concretes after only a few days of drying at ordinary temperature. Test specimens of relatively small thickness may be dried without the formation of shrinkage cracks.

For uncracked insulating concretes, the ratio of flexural to compressive strength may be higher than for structural concretes and highest for the materials of lowest compressive strength. Approximate ratios of flexural strength to compressive strength for some insulating concretes (75 to 500 psi compressive strength) are as follows:[2,5,6,8,10,15,16]

Aggregate	Ratio of flexural to compressive strength
Perlite	0.20 to 0.45
Vermiculite	0.25 to 0.50
Expanded slag	0.30
Expanded shale	0.30
Sand (cellular)	0.20 to 0.35

Ratios lower than 0.1 reported in the literature for some concretes made with Group I aggregates, and pumice, and for cellular neat cement are believed to have been obtained upon cracked specimens.

Approximate ratios of bond strength, as determined for pull-out specimens with embedded reinforcing bars, to compressive strength (for compressive strengths under 500 psi), are reported as follows:

Aggregate		Ratio of bond strength to compressive strength
Perlite		0.15 to 0.23
Vermiculite		0.35 to 0.64
Pumice	aerated no-fines	0.25 to 0.50
Expanded slag	aerated no-fines	
Expanded shale	aerated no-fines	
Sand (cellular)		0.25 to 0.38
None (cellular neat cement)		0.25

These values, from several different sources,[2,8,15,16] are not necessarily comparable because of differences in types of specimens and reinforcing bars.

Elasticity

"Initial tangent," "secant," and dynamic moduli of elasticity values have been reported for insulating concretes.[2,5,6,8,15,16] Ranges in compressive secant moduli were as follows for Group I and Group III concretes:*

Aggregate, proportions	Compressive strength, psi	Secant modulus, 1000 psi
Perlite, 1:8 to 1:4	80 to 450	70 to 250
Vermiculite, 1:8 to 1:3	70 to 370	40 to 140
Sand (cellular), 1:3	150 to 550	150 to 700
None (cellular), 1:0	85	70

*The slope of the secant drawn through the origin and point on stress-strain curve at one-half the ultimate stress is one definition of the secant modulus.

The above-listed values are approximate. Data were obtained for specimens 28 to 60 days old.

Secant moduli for concretes made with pumice, expanded shale, and expanded slag aggregate, and having 28-day compressive strength of about 500 psi, have ranged from 700,000 to 900,000 psi. For air-entrained (25 to 35 percent) no-fines concretes made with the same aggregates, secant moduli values were approximately 1000 times the compressive strength values in the 28-day compressive strength range of 160 to 530 psi.

Water absorption of insulating concretes

The 24-hr room-temperature water absorptions, by volume, of oven-dry Group I concretes have been reported as follows:[5,6,10,15] perlite-aggregate concrete, 18 to 48 percent, and vermiculite-aggregate concrete, 27 to 51 percent.

Values reported by Kluge, Sparks, and Tuma[6] increased with the proportion of perlite or vermiculite in the concrete. Values may be lower than 30 percent for highly air-entrained 1:6 perlite concrete and somewhat higher for similarly proportioned vermiculite concrete.

Average absorption values for Group II and cellular concretes are reported in the adjoining table.[2,8,18,21]

Aggregate	Absorption, percent	
	By weight	By volume
Pumice	25–28	18–29
Expanded slag	23	24
Expanded slag (no-fines, aerated)	13–16	12–15
Expanded shale	23	22
Expanded shale (no-fines, aerated)	9–15	9–16
Sand (cellular)	32	28
None (cellular neat cement)	25–43	20–35

Rate of drying

A search of the literature failed to reveal any comprehensive study of the moisture content and rate of drying of insulating concretes subjected to strictly controlled ranges of temperature and humidity. The drying rates of small perlite- and vermiculite-aggregate concrete slabs 3 in. thick have been separately reported.[15,16] These tests were made in "laboratory air" of unstated humidity. Lund[26] has reported on water vapor diffusion and moisture migration through insulating perlite concrete used in built-up roof decks.

These studies are admirable recognition of a most serious problem with insulating concretes and, with some refinements and closer control of test conditions, might well contribute to formulating job practices to eliminate moisture in dangerous amounts in applications requiring early waterproofing of insulating concretes.

Resistance to freezing and thawing

Although the usual application of insulating concrete does not require that the material withstand frost action, several freezing and thawing studies of Group I and Group II concretes have been reported.[5,6,8,15] These studies differed in types of specimens, freezing and thawing cycles, and criteria for resistance. It may be concluded, however, that the relatively high air contents of certain of these concretes were apparently effective in resisting frost action. Some of the vermiculite and perlite mixtures were among those showing the greatest resistance to freezing and thawing among a number of lightweight aggregate concretes.[5] Other perlite and vermiculite mixes, especially those containing the highest proportions of aggregate, were relatively poor in resistance to freezing and thawing. No-fines concretes made with Group II aggregates and containing less than 30 percent of entrained air usually withstood 100 cycles of a severe rapid freezing and thawing test.[8]

Drying shrinkage

Group I, pumice-aggregate, and moist-cured cellular concretes exhibit relatively high drying shrinkages.[2,5,6,8]

For Group I concretes and cellular concretes containing sand, some studies showed that shrinkage increased with the ratio of cement to aggregate.

Shrinkage movements for some perlite concretes have at times been reversed at later ages. This phenomenon has yet to be satisfactorily explained.

Aggregate	Percent shrinkage, 6 month drying
Perlite	0.10 to 0.23
Vermiculite	0.17 to 0.47
Pumice	0.10 to 0.23
Expanded slag	0.06 to 0.10
Expanded shale	0.06 to 0.11
Sand (cellular)	0.10 to 0.50
None (cellular neat cement)	0.48 to 0.66

Thermal expansion

Coefficients of thermal expansion of insulating concretes are reported in the table below.[5,15,16,18,27]

Aggregate	Coefficient of thermal expansion, per deg F
Perlite	4.2 to 6.1×10^{-6}
Vermiculite	4.6 to 7.9×10^{-6}
Pumice	5.2 to 6.0×10^{-6}
Expanded slag	5.1 to 6.2×10^{-6}
Expanded shale	3.6 to 4.5×10^{-6}
Moist-cured neat cement (noncellular)	6.5×10^{-6}

THERMAL CONDUCTIVITY

Oven-dry insulating concretes

Average thermal conductivity values for oven-dry insulating and other concretes, as reported by a number of sources,[2,5,6,28-31] are presented in Table 3. Some differences between values from different sources may be expected, especially for tests made at different mean temperatures. Some of the data from which the tabulated averages were derived indicate that the conductivity may be as much as 15 percent higher at a mean temperature of 120 F than at 32 F.

TABLE 3—AVERAGE THERMAL CONDUCTIVITY VALUES FOR OVEN-DRY INSULATING AND OTHER CONCRETES

Density, oven-dry, lb per cu ft	Average thermal conductivity, Btu per hr per sq ft (deg F per in.), at indicated mean temperature						
	HHFA,* 120 F	Rowley and Algren,† 75 F	Kreuger and Eriksson‡	Cammerer‡	Kudriashev, § 70 F	Watzinger,‡ 70 F	Valore,¶ 120 F
15	—	—	0.4	0.4	—	0.5	—
20	0.70	—	0.5	0.45	—	0.5	0.65
30	0.90	—	0.75	0.55	0.9	0.7	—
40	1.15	—	1.05	0.75	1.1	0.9	1.2
50	1.4	—	—	—	1.35	1.15	1.5
60	1.7	1.8	1.8	1.2	1.7	1.45	1.85
70	2.1	—	—	—	—	—	2.4
80	2.5	2.75	2.8	1.85	—	—	3.1
90	3.0	—	—	—	—	—	—
100	3.6	4.0	4.35	2.8	—	—	4.65
120	5.4	6.6	6.75	4	—	—	6.7
140	8.6	10.5	11	6.1	—	—	9.2

*HHFA[7] values are from average curve derived from data of Kluge, Sparks, and Tuma[6] and Price and Cordon[5] for lightweight aggregate concretes.

†Rowley and Algren's[28] values were obtained for dense- and lightweight-aggregate concretes having 2.5 to 12.8 percent free moisture; the values presented here were corrected by present author to oven-dry condition by assuming that an increase in density due to an increase of 1 percent in free moisture causes an increase in conductivity of 5 percent.

‡Cellular concrete and other inorganic building materials.[29-31]

§Autoclaved cellular concrete.[32]

¶Ordinary dense concrete, no-fines air-entrained dense and lightweight-aggregate concretes and cellular concretes.[2]

Concretes of the same density, but containing different Group II lightweight aggregates, showed large differences in conductivity, as reported by Kluge, Sparks, and Tuma.[6] At a density of 85 lb per cu ft, for example, the conductivity values of concretes containing an expanded clay or an expanded slate were 70 percent higher than those for concretes containing an expanded slag. Values reported more recently by the Housing and Home Finance Agency[19] for expanded slag, expanded shale, and pumice concretes having densities in the range 51 to 86 lb per cu ft, are not significantly different from the average conductivity-density relationship derived earlier by the HHFA.[7] Values obtained at the National Bureau of Standards[8] for lightweight- and dense-aggregate no-fines concretes containing 20 to 35 percent of entrained air were generally appreciably higher than those for conventional lightweight aggregate concretes of similar density reported by Kluge, Sparks, and Tuma,[6] as indicated in Fig. 1. According to Rowley and Algren,[28] the flow of heat through a heterogeneous mass composed of ingredients of unequal conductivities occurs mainly through the elements of higher conductivity. This sug-

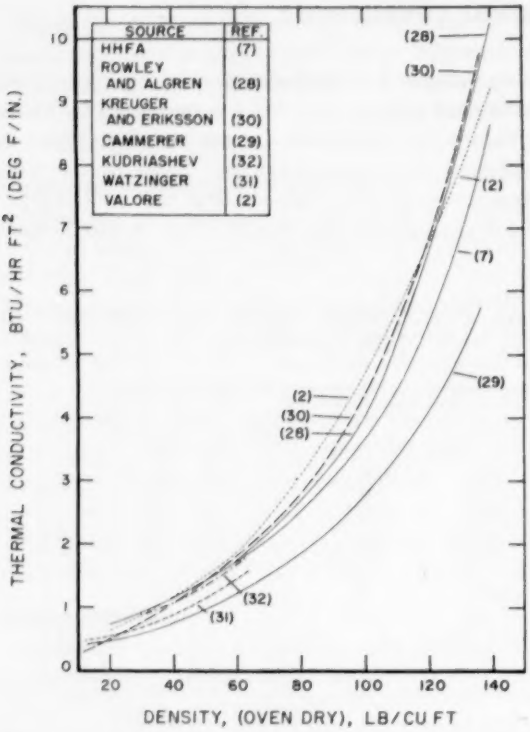

Fig. 1—Average relationships of thermal conductivity to density. Curves from references 28 and 7 are for lightweight- and dense-aggregate concretes; 29-32 are for cellular concretes, dense concretes, and "inorganic building materials;" 2 is for no-fines air-entrained lightweight- and dense-aggregate concretes, conventional concrete, and cellular concretes

gests an influence of the size and density of coarse aggregate on the disparate values that have been obtained for no-fines air-entrained dense aggregate concretes and Group II-aggregate concretes of the same density.

Influence of moisture on thermal conductivity

The current practice of determining and reporting conductivity values for materials in an oven-dry condition is not entirely satisfactory for indicating insulating value of concrete in service. Different concretes in equilibrium with surroundings at a given temperature and relative humidity may contain significantly different percentages of free moisture.* The use of oven-dry specimens is an expedient dictated both by uncertainty concerning the actual moisture content of the concrete in service and experimental difficulties encountered in hot-plate conductivity tests of materials containing appreciable amounts of moisture.

Conductivity values for cellular concretes containing different percentages of free moisture have been reported by Kudriashev[32] and Graf;[33] values for limerock-aggregate concrete have been determined by Tyner.[34] Average effects of free moisture, reported as the percentage increase in thermal con-

*The percentage of free moisture in insulating concrete is defined as 100 times the ratio of the loss in weight upon oven-drying at approximately 220 F to the oven-dry weight.

ductivity for each percent increase in density due to free moisture, were as follows:

Type of concrete	Density, lb per cu ft	Source	$\Delta k/\Delta d$
Lime-silica autoclaved cellular concrete	31 to 62	Kudriashev[32]	4
Various cellular concretes	24 to 84	Graf[33]	6
Expanded shale concrete	61	Petersen[18]	4
Limerock concrete	97	Tyner[34]	5

The $\Delta k/\Delta d$ values shown represent the percentage increase in conductivity for each increase of 1 percent in weight due to free moisture. These values are for temperature "above freezing;" for subfreezing temperature Kudriashev[32] reported corresponding values about double those shown.

Tyner[34] cites corresponding values for concrete, obtained by others, ranging from 3 to 11. Of 18 values reported by Graf[33] 16 were between 4 and 7.8; the others were 10 and 17. Where notably high values are reported the possibility exists that appreciable moisture migration occurred during test and partially influenced the result. However, the frequency with which values between 4 and 6 occur suggests that at least an approximate value for the conductivity of concrete containing free moisture may be estimated by increasing the conductivity value obtained for oven-dry concrete by 5 percent for each percent increase in density due to free moisture.

Petersen[18] reported values for thermal conductivity of specimens 1 in. thick, and of walls 8 in. thick, made from expanded shale and expanded slag concretes having dry densities of 61 to 94 lb per cu ft. Thermal conductivity values derived from the test data for the 8-in. walls were between 30 and 60 percent greater than those obtained for the nearly-dry 1-in. specimens. Petersen[18] attributed the higher values obtained for the thicker specimens to an appreciably higher, but not definitely known, moisture content.

FIRE RESISTANCE

Resistance of concrete to the flow of heat is an important factor affecting fire resistance. Thermal expansion and the chemical and mechanical stability of a concrete may also affect its fire resistance. The fire-protective function of concrete ranges from that of supporting loads during and after a fire, to that of serving as expendable, insulative protection for structural steel from high temperature during a fire. The latter function will be discussed here. Carlson[35] discusses fire resistance of walls of masonry units.

Heat is transferred through concrete exposed to fire by conduction and, at high temperature, by radiation. The latter will depend upon the number of air-solid interfaces traversed through a given thickness of material. Lightweight concretes, either cellular or containing lightweight aggregate that is

itself cellular in structure, offer good resistance to flow of heat due to radiation. The initial high moisture content of some lightweight concretes may also offer significant temporary advantage.[36,37]

Fire tests are generally performed upon relatively large-size models of wall, floor, and column constructions. Because each test is costly, data are not sufficiently abundant to fall into simple classification solely on the basis of type of lightweight aggregate or concrete used in the test structures.

Estimated fire resistance ratings of walls and partitions of hollow concrete masonry units are reproduced by the Portland Cement Association[38] from the National Building Code of 1949.[39] These ratings are given in terms of the "equivalent" or average thickness of solid material in a wall required for 1-, 2-, 3-, and 4-hr fire resistance. For a given rating the required thickness was least for units of expanded slag or pumice. Next, in order of increasing thickness, were expanded clay or shale; limestone, cinders, or unexpanded slag; calcareous gravel; and siliceous gravel.

Fire tests have been performed on walls of reinforced concrete with perlite[40] and vermiculite[17] aggregates and reinforced moist-cured[41] and autoclaved[32,42] cellular concrete; a non-load-bearing perlite-aggregate solid masonry wall has also been fire-tested.[43] Because of differences in these wall constructions, test results are not directly comparable with each other or with those reported for walls of units with Group II aggregates. However, these few results indicate that non-load-bearing walls of reinforced concrete or concrete masonry units containing perlite or vermiculite aggregate, and reinforced cellular neat cement, may attain 4-hr fire resistance ratings in moderate wall thicknesses. Good ratings also appear attainable for these materials in load-bearing walls but the required thickness appears to depend upon the amount and nature of reinforcement and the use of mixes having appreciably greater density and structural value than are ordinarily required in roof-fill applications.

Fire tests of steel and reinforced concrete columns encased in concretes made with pumice, cinders, and certain dense aggregates have been reported by Mitchell.[37]

Fire resistance ratings of steel columns protected by gypsum plaster containing sand, perlite, and vermiculite aggregates are presented by Crosby, Fiske, and Forster.[44] Perlite and vermiculite appear to offer potential advantage over dense siliceous sand as ingredients in fire-protective materials for structural elements.

ACOUSTIC PROPERTIES

The properties sound insulation and sound absorption are distinctly different entities.* A wall is said to possess good sound insulating properties if the intensity of air-borne sound energy incident upon one side of the wall is attenuated or reduced to a satisfactory degree upon passing through the wall.

*This paragraph is largely based on discussions of the subject by the British Building Research Station,[45] Knudsen and Harris in *Acoustical Designing in Architecture*,[47] and by the National Bureau of Standards.[48,49]

Sound absorption, however, refers to the degree of "nonreflection" exhibited by the surfaces against which sound energy impinges; therefore, sound absorbent materials are not sound insulating in the direct sense. An open window is often employed as a standard of 100 percent absorption (0 percent reflection) but also is virtually 100 percent transmitting (noninsulating). The sound insulation properties of the walls of a room reduce noise in the room caused by air-borne sound originating outside the room; sound absorbent treatment of the walls or ceiling of a room reduces the duration and intensity of reverberant or severally reflected sound originating within the room.*

Sound insulation

The problems of insulating air-borne and solid-borne noise are quite different, according to Knudsen and Harris:[47]

> "Although massive, rigid partitions (for example, concrete walls) provide effective insulation against air-borne noise, they offer poor protection against solid-borne vibration."

Mechanical vibration may be transmitted along solid paths with little attenuation. Thus, a bare concrete floor may readily transmit the sound of footsteps to a room below. Solid-borne noise should be suppressed at its source, according to Knudsen and Harris; otherwise, marked discontinuities in density or elasticity in the solid transmission path are required to reduce sound transmission.

The measure of sound insulation is the "transmission loss," expressed in "decibel" units:

$$\text{Transmission loss (TL)} = 10 \log_{10} (E_i/E_t)$$

where E_i is the level of sound energy incident upon a wall or partition, and E_t is the level of sound energy transmitted through the wall. A transmission loss of 10 db (decibels) indicates that 1/10 of the incident sound energy is transmitted; 20 db, 1/100 is transmitted; 30 db, 1/1000, and so on.† The entire range of human audibility is about 120 db, or about 10^{12} times the least audible sound. Sound intensities above 120 db are perceived as pain.

The degree of insulation of air-borne sound considered adequate will vary with the type of occupancy, the external noise level, and with the individuals occupying a building. Recommended minimum TL values for party walls between apartments and for walls for hospital rooms, range from 45 to 55 db in Canadian,[50] Swedish,[47] and British[51,52] building codes, and according to recommendations of the American Public Health Association.[53] Values as low as 40 db may be satisfactory for school classrooms.

According to various studies[47,48,52-54] of the insulation of homogeneous‡

*Knudsen and Harris[47] state that the level of noise which is transmitted into a room is reduced relatively little by sound absorbent treatment.

†Values reported for transmission loss are usually averages of individual values obtained for a number of different frequencies that range from about 100 to 4000 cps. TL values for concrete are usually higher at the highest of these frequencies and the reported average value may be several db lower if a smaller frequency range (*e.g.*, 100 to 2000 cps) was covered.

‡For the purposes of this discussion of sound insulation a wall is considered "homogeneous" if both sides vibrate together. In this sense, plastered walls of concrete or clay masonry units, and partitions of plaster on metal or other lath are homogeneous.

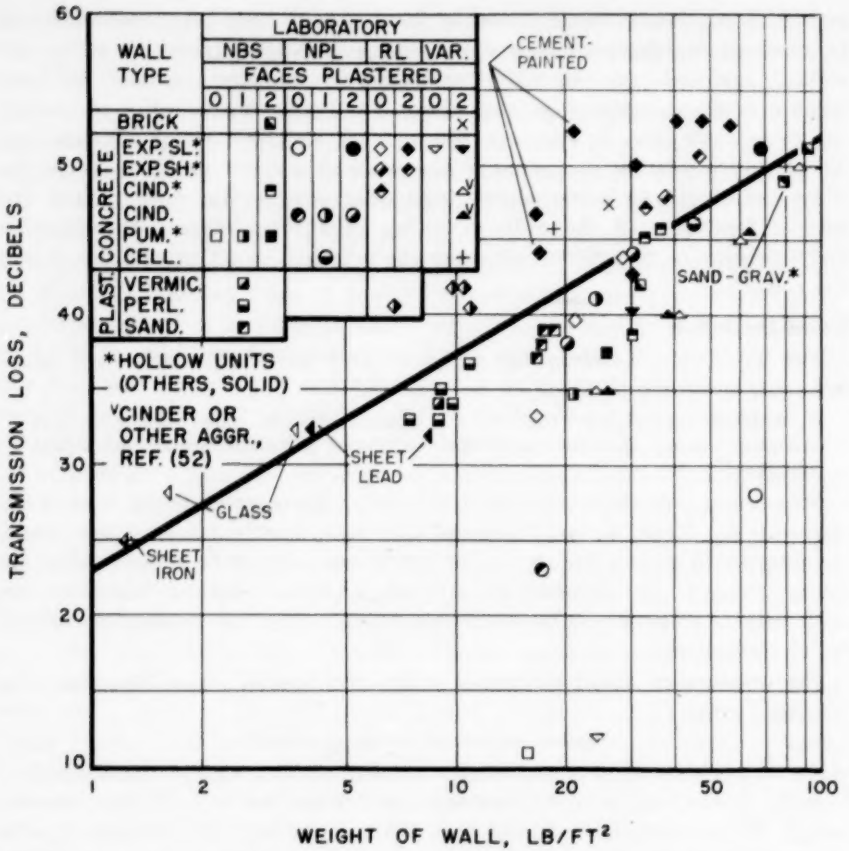

Fig. 2—Average transmission loss data for air-borne sound for concrete masonry and other types of walls. Tests conducted by National Bureau of Standards, National Physical Laboratory of Great Britain, Riverbank Laboratories, and various other laboratories. Transmission loss values are averages for frequency ranges of approximately 100 to 2000, 3000, or 4000 cps. Heavy line is average transmission loss-weight relationship of Knudsen and Harris, reference 47

porous walls against air-borne sound, the weight per unit of wall area is the most important factor influencing the transmission loss. The stiffness, manner of attachment at edges, and nature of the materials are of secondary importance. Fig. 2 contains TL and weight data for walls of concrete and plaster in which lightweight and other aggregates were used. The heavy straight line represents the average relationship between TL and weight presented by Knudsen and Harris,[47] and indicates that the effect of doubling the weight of a homogeneous wall is to increase the TL by about 4.5 db. Differences smaller than 3 or 4 db in values from different laboratories should not be regarded as significant. Knudsen and Harris state that plastered walls of

TABLE 4—EFFECT OF PLASTERING OR PAINTING ON SOUND INSULATION OF WALLS OF CONCRETE MASONRY

Type of concrete unit and aggregate	Reference*	Treatment	Weight, lb per sq ft	TL, db
4-in. hollow pumice block	48	None	15.5	11.0
		Plastered, one face	20.4	34.6
		Plastered, two faces	25.3	37.4
Hollow expanded slag block filled with sand	47	None	65	28
		Plaster skim coat, both faces	68	51
4-in. hollow expanded slag block	56	None	23	12
		Acoustical plaster both faces	28	40
8-in. hollow expanded slag block	38	None	28.6	43.7
		Plastered, both faces	40	52.9
4-in. hollow expanded slag block	38	None	16.5	33.2
		Plastered, both faces	31.0	50.0
		Cement paint both faces	16.5	46.7
6-in. hollow expanded slag block	38	None	21	39.7
		Cement paint both faces	21	52.2
3-in. solid cinder slabs	47	Plain	17	23
		Plastered one face	24	41
		Plastered both faces	30	44

*Tests reported in numbered references were conducted in the following laboratories:
Reference 38, Riverbank Laboratories.
References 47, National Physical Laboratory of Great Britain.
References 48, National Bureau of Standards.

porous building block usually have TL values 3 or 4 db higher than those predicted by the straight-line relationship.

Porous, rigid materials may be expected to follow the weight-TL relationship only if the pores are not continuous. In some types of block made with harsh, dry concrete, the TL will be much lower than predicted from weight alone because some transmission of air-borne sound will occur through the continuous air paths in the material. Table 4 indicates a marked effect of plastering, or painting with cement-water paint, in increasing the transmission loss of walls of blocks or slabs made of lightweight aggregate concrete.

Most of the data available on air-borne sound insulation of lightweight concrete are for walls of concrete masonry units. Fig. 2 indicates wide divergences for data from different laboratories. Results obtained in each of the laboratories giving values different from those predicted by the Knudsen and Harris[47] straight line are themselves more or less consistent in following a weight relationship of approximately the same slope as, but as much as 5 db higher or lower than, the presented relationship.

No satisfactorily comprehensive investigation of the sound insulation value of concretes made with all of the most widely used lightweight aggregates has yet been made.

There is no indication that lightweight aggregates because of their "lightness" or for any other reason, impart special sound insulation properties to concrete. On the contrary, for solid constructions, and even for hollow masonry walls, increasing the weight of the wall generally improves the sound insulation value of the wall. Values presented for perlite and ver-

miculite are limited largely to plasters containing these aggregates and comparisons with sanded plasters, but concretes containing these aggregates would probably follow fairly generally the Knudsen and Harris[47] relationship.

If single, rigid partitions are to be used, the TL obtained cannot exceed appreciably the value given by the TL-weight relationship shown in Fig. 2, *e.g.*, if 50 db TL is required of a single rigid partition, its weight must be about 70 lb per sq ft. Fortunately, however, architects and acousticians have devised special types of discontinuous construction, resilient clips for attaching metal lath, etc., which permit the attainment of good sound insulation at relatively low weight. The reader interested in devices of these types, which are quite outside the scope of the present report, is referred to the work of Knudsen and Harris,[47] and of the National Bureau of Standards.[48]

Sound absorption

The most commonly used absorption coefficient is a number expressing the ratio of the amount of sound energy absorbed (not reflected) by a surface, to the amount of energy incident upon that surface. Thus, an absorption coefficient of 0.03, a value commonly obtained for smooth plaster or for a smooth, trowelled or cast concrete surface, indicates that 97 percent of the incident sound energy is reflected by the surface. Absorption values are usually given for a range of sound frequencies from about 125 to 2000 cps (cycles per sec.); absorption of 500 cps sound is the criterion usually employed for sound absorbents for auditoria. The average absorption at a number of frequencies from 125 to 2000 cps is sometimes termed the "noise reduction coefficient."

The sound absorption properties of materials may reside largely in the character of their surfaces exposed to sound energy. Porosity of surface is generally needed for good sound absorption, but the pores should be interconnected from the surface to the interior of a material so that the sound energy may impinge upon a large, internal, porous surface. When this condition obtains, the absorption will depend also on the thickness of the material and nature of the backing.

Typical absorption coefficients (at 500 cps) and noise reduction coefficients for concretes containing various aggregates, as reported by Watson and Morrical,[55] the British Building Research Station,[56] and other sources, are presented in Table 5.

Although relatively good sound absorption may be obtained for lightweight aggregate concrete having a porous surface, values almost as good may be obtained for similarly porous dense aggregate concrete.[38] However, the desirably porous surfaces, especially in concrete block, are more easily attained with many of the lightweight aggregates than with dense aggregates. Vermiculite and perlite aggregates have been used extensively as ingredients in sound absorbent plasters.

Inasmuch as sound absorption depends upon the nature of the concrete surface, untreated surfaces are much more absorbent than painted or plastered

TABLE 5—TYPICAL SOUND ABSORPTION DATA FOR VARIOUS CONCRETES AND PLASTERS

Material	Reference	Absorption coefficient, 500 cps	Noise reduction coefficient, 125 to 2000 cps
Perlite acoustical plaster	58	0.4	0.55
1:8 vermiculite concrete	16	0.8	0.7
Lightweight aggregate block	38	0.2–0.7	—
Dense aggregate block	38	0.2–0.6	—
Expanded slag block	21	—	0.4–0.6
Expanded slag block	57	0.4	0.35
Cinder block mixes	55	0.15–0.8	—
Expanded shale block mixes	55	0.2–0.8	—
Sand-gravel block mixes	55	0.1–0.4	—
Limestone aggregate block mixes	55	0.1–0.6	—
Expanded shale aggregate block	20	0.6	0.7
Cinder aggregate concrete	46	0.6	0.4
Concrete block (dense)*	52	0.02	0.02
Concrete block (lightweight)*	52	0.2–0.3	0.2–0.35
Smooth plaster (solid backing)	46	0.02	0.03
Concrete (smooth trowelled finish)	46	0.02	0.02

*Remote side of block sealed.

surfaces. Three or four coats of cement-water paint lowered the sound absorption of porous concrete surfaces rather markedly in the study reported by Watson and Morrical.[55] Smooth plaster almost nullifies the absorptive value of porous concrete block. This fact illustrates again how decidedly different are the factors upon which sound insulation and sound absorption depend. Concrete porous enough to be a good absorbent will usually have poor insulation value. The virtual elimination of good absorptive properties by plastering the concrete will usually result in markedly improved sound insulation value, as indicated in Table 4.

No study of the sound absorption of cellular concretes has been reviewed in preparing the present report. However, because the pores or cells of cellular concrete are generally not intercommunicating, high values of sound absorption might not be expected for this type of material.

RECOMMENDATIONS

Extant specifications for concretes for thermal insulation are in a formative state, and show either inadequacies or unnecessary restrictions. This is unavoidable, because of the relative newness of some of these materials and reliance has been placed upon methods applicable to more conventional structural concretes.

Future research on insulating concretes might consider some of the following subjects:

1. Modification of aggregate grading requirements.
2. Choice of satisfactory test specimen fabrication techniques.
3. Choice of a satisfactory compressive strength test specimen from standpoint of size, shape, and method of capping.

4. Investigation of effect of oven drying on strength of specimens of different sizes.
5. Significance of indentation strength.
6. The advisability of performing briquette tensile tests.
7. Drying rates in controlled atmospheres.
8. Dimensional stability over long periods and its importance.
9. Applicability of cellular concretes.

REFERENCES

1. Foster, B. E., "Absorption by Concrete of X-Rays and Gamma-Rays," ACI JOURNAL, Sept. 1953, *Proc.* V. 50, pp. 45-64.

2. Valore, R. C., Jr., "Cellular Concretes," ACI JOURNAL, May and June 1954, *Proc.* V. 50, pp. 773-796 and 817-836.

3. "Lightweight Aggregates for Insulating Concretes," ASTM Tentative Specification C 332-54T.

4. Washa, G. W., "Properties of Lightweight Aggregates and Lightweight Concretes," ACI JOURNAL, Oct. 1956, *Proc.* V. 53, pp. 375-382.

5. Price, Walter H., and Cordon, William A., "Tests of Lightweight-Aggregate Concrete Designed for Monolithic Construction," ACI JOURNAL, Apr. 1949, *Proc.* V. 45, pp. 581-600.

6. Kluge, R. W., Sparks, M. M., and Tuma, E. C., "Lightweight-Aggregate Concrete," ACI JOURNAL, May 1949, *Proc.* V. 45, pp. 625-644.

7. "Lightweight Aggregate Concretes," Housing and Home Finance Agency, U. S. Government Printing Office, Washington, D. C., 1950, 28 pp. This booklet summarizes and discusses references 3 and 4.

8. Valore, R. C., Jr., Unpublished data of the National Bureau of Standards, Washington, D. C., 1955.

9. "Lightweight Aggregates for Structural Concretes," ASTM Tentative Specification C 330-53T.

10. Brouk, J. J., "Perlite Insulating Concrete," ACI JOURNAL, June 1954, *Proc.* V. 50, pp. 857-867.

11. Willson, Cedric, Private communication, Texas Industries, Inc., Dallas, May 1955.

12. "The Mearlcrete System for Foam Concrete," Mearl Manufacturing Corp., New York, 1954.

13. "National Concrete Foam Liquid and Apparatus," National Foam System, Inc., West Chester, Pa., 1953.

14. "A New Development in Building," Cellular Products Division, Pyrene Co., Ltd., Brentford, Middlesex, England, 29 pp.

15a. "New Data on Concrete Aids Designers," *The Perlite Torch*, V. 3, No. 2, 1953, p. 1.

15b. Walker, C. G., Private communication, Atlantic Perlite Co., Washington, D. C., 1955.

16. "Zonolite Concrete," Universal Zonolite Insulation Co., Chicago, 1941, 86 pp.

17. Spence, J. C., Private communication, Vermiculite Institute, Chicago, Sept. 1954.

18. Petersen, P. H., "Burned Shale and Expanded Slag Concretes With and Without Air-Entraining Admixture," ACI JOURNAL, Oct. 1948, *Proc.* V. 45, pp. 165-176.

19. "Design Data for Some Reinforced Lightweight Aggregate Concretes," *Housing Research Paper* No. 26, Housing and Home Finance Agency, Washington, D. C., Oct. 1953, 19 pp.

20. Erskine, F. G., Private communication, Expanded Shale Institute, Washington, D. C., 1954.

21a. Bauman, E. W., Private communication, National Slag Association, Washington, D. C., 1954.

21b. "Slag Concrete Masonry Units," National Slag Association, Washington, D. C., 1954.

22. Copeland, L. E., and Bragg, R. H., "Self-Desiccation in Portland Cement Pastes," ASTM *Bulletin* No. 204, Feb. 1955, pp. 34-39.

23. Tucker, Jr., John, "Effect of Length on the Strength of Compression Test Specimens," *Proceedings*, ASTM, V. 45, 1945, p. 976.

24. Bessey, G. E., and Dilnot, S., "The Relation Between Strength and Free Water Content of Aerated Concretes," *Magazine of Concrete Research* (London), No. 3, Dec. 1949, p. 119.

25. "Guide Specification for Military Construction; Concrete (For Building Construction)," Department of the Army, Corps of Engineers, Office of the Chief of Engineers, CE-204 (INT), Sept. 15, 1953, p. 19 and p. 29.

26. Lund, C. E., "Vapor Diffusion and Moisture Migration Through Insulating Perlite Concrete Used in Built-Up Roof Decks," Perlite Institute, New York, 1954, 43 pp.

27. Menzel, C. A., "Strength and Volume Change of Steam-Cured Portland Cement Mortar and Concrete," ACI JOURNAL, Nov.-Dec. 1934, *Proc.* V. 31, pp. 125-148.

28. Rowley, F. B., and Algren, A. B., "Thermal Conductivity of Building Materials," *Bulletin* No. 12, University of Minnesota Engineering Experiment Station, 1937, 134 pp.

29. Cammerer, J. S., *Die Konstruktive Grundlagen des Warme- und Kalteschutzes in Wohn- und Industriebauten* (Principles of Construction for Protection Against Heat and Cold in Residences and Industrial Buildings), Julius Springer, Berlin, 1936.

30. Kreuger, H., and Eriksson, A., "Undersokningar rorande byggnadskonstruktioners varmeisolerings formaga II (Investigation of Heat Insulation Properties of Building Constructions)," *Handlingar* (*Proceedings*) No. 36, Royal Swedish Institute for Engineering Research, Stockholm, 1936.

31. Watzinger, A., "Varmeledmingstall for Byggematerialer (Thermal Conductivity Values of Building Materials)," *Report* No. 1, Norwegian Building Research Institute, Oslo, 1950.

32. Kudriashev, I. T., "Autoclaved Cellular Concrete," Central Scientific Institute for Building Materials, State Publishers of Literature on Construction, Moscow, 1949, 188 pp.

33. Graf, O., *Foam Concrete, Gas Concrete, Light Lime Concrete*, Konrad Wittwer, Stuttgart, Germany, 1949, 76 pp.

34. Tyner, Mack, "Effect of Moisture on Thermal Conductivity of Limerock Concrete," ACI JOURNAL, Sept. 1946, *Proc.* V. 43, pp. 9-20.

35. Carlson, C. C., "Lightweight Aggregates for Concrete Masonry Units," ACI JOURNAL, Nov. 1956, *Proc.* V. 53, pp. 491-508.

36. Foster, H. D., Pinkston, E. R., and Ingberg, S. H., "Fire Resistance of Walls of Lightweight-Aggregate Masonry Units," *Building Materials and Structures Report* BMS 117, National Bureau of Standards, Washington, D. C., May 1950, 23 pp.

37. Mitchell, N. D., Private communication; unpublished data of the National Bureau of Standards, Washington, D. C., 1952.

38. *Concrete Masonry Handbook*, Portland Cement Association, Chicago, 1951, 64 pp.

39. *National Building Code*, Recommended by National Board of Fire Underwriters, New York, 1949 Edition, 258 pp.

40. "Report on 'Bildrok' 6½-in. Steel Reinforced Perlite-Portland Cement Bearing Wall," Underwriters Laboratories Inc., Chicago, Retardant 3410-1, 1952.

41. "Report on 8-in. Load-Bearing Walls of Reinforced Higgins 'Thermo-Con'," Underwriters Laboratories, Inc., Chicago, Retardant 3208, 1951.

42. Ahlstedt, E., Private communication, International Ytong Co., Stockholm, Sweden, 1955.

43. *Fireproofing with Perlite*, 4th Edition, Perlite Institute, New York.

44. Crosby, E. U., Fiske, H. A., and Forster, H. W., *NFPA Handbook of Fire Protection*, National Fire Protection Association, Boston, 11th Edition, 1954, 1560 pp.

45. Underwriters Laboratories Inc., National Board of Fire Underwriters, Chicago (Fire tests of various walls of lightweight-concrete masonry units; reported separately).

46. "Sound Absorbent Treatments," *Building Research Station Digest* No. 36, Nov. 1951, H. M. Stationery Office, London, 10 pp.

47. Knudsen, V. O., and Harris, C. M., *Acoustical Designing in Architecture*, John Wiley and Sons, New York, 1950, 457 pp.

48. "Sound Insulation of Wall and Floor Constructions," *Building Materials and Structures Report* No. 144, National Bureau of Standards, Washington, D. C., Feb. 1955, 66 pp.

49. London, A., "Noise Reduction in Dwellings," *Architectural Record*, Oct. 1949, pp. 141-145.

50. *National Building Code of Canada*, "Use and Occupancy," Part 3, Associate Committee on the National Building Code, National Research Council, NRC No. 3191, Ottawa, Canada, 1953, (Section 3.13, "Acoustical Insulation").

51. Parkin, P. H., and Stacy, E. F., "Recent Research on Sound Insulation in Houses and Flats," *Journal of the Royal Institute of British Architects*, Third Series, V. 61, No. 9, July 1954, p. 372.

52. "Walls and Partitions," British Standard Code of Practice CP (1950) General Series, Draft Code 122, Council for Codes of Practice for Buildings, London, 1950, 119 pp.

53. Bolt, R. H., "Control of Noise," *Construction and Equipment of the Home*, American Public Health Association Committee on the Hygiene of Housing, Public Administration Service, Chicago, 1951, p. 33.

54. "Aerated Concrete," Part I, "Properties," and Part II, "Applications," *Building Research Station Digest*, No. 28 and No. 29, Mar. and Apr. 1951, H. M. Stationery Office, London, 4 pp. and 6 pp.

55. Watson, F. R., and Morrical, K. C., "Sound Absorbing Value of Portland Cement Concrete," ACI JOURNAL, May-June 1936, *Proc.* V. 32, pp. 659-672.

56. "The Reduction of Sound Transmission Through Concrete Floors," *Building Research Station Digest*, No. 19, June 1950, H. M. Stationery Office, London, 6 pp.

57. Foran, M. R., Ball, S., Johnson, R. E., and Wallace, J. R., "Sydney Foamed Slag as Lightweight Aggregate," *The Engineering Journal* (Montreal), May 1953, p. 2.

58. "Annealed Perlite Aggregates," F. E. Schundler and Co., Joliet, Ill.

For such discussion of this paper as may develop please see June 1957 JOURNAL.

Title No. 53-28

Prestressed Bridge Designed for Crane Load at Niagara River Weir*

By A. M. LOUNT†

SYNOPSIS

Cranes which operate gates of a control weir on the Niagara River cause heavily eccentric live loading on the 1512-ft supporting bridge. Bridge designer selected a homogeneous grid system of longitudinal girders tied together by diaphragms, covered with a 7-in. slab acting compositely with the longitudinal girders. All components of the grid are prestressed. Design considerations of transverse bending stiffness and torque rigidity are discussed, and step-by-step method for exact structural analysis of the grid is summarized. Construction features are treated briefly with emphasis on shear connectors, stirrup spacing, and prestressing system.

INTRODUCTION

Extensive remedial works designed to preserve the beauty of Niagara Falls are currently being undertaken jointly by the Ontario Hydro-Electric Power Commission and the United States Corps of Engineers. Part of these works is a control weir on the Canadian side of the river just above the falls. The gates in this weir are serviced by cranes operating from a bridge deck which spans the openings of the weir, and as a result the loads involved are quite unusual, both in size and in nature. The total live load is in excess of 100 tons, and this can be heavily eccentric.

Due to restrictions imposed by the construction schedule, only a minimum of time could be allowed for construction. Because of this and for other, aesthetic reasons, the use of precast prestressed concrete was adopted. Prestressing also permitted the construction of a light and completely homogeneous grid with many of the properties of a solid slab. This was of importance due to the high eccentricity of the load.

GENERAL DESCRIPTION

Fig. 1 shows a general view during construction of the 1512-ft structure, and a side view of the bridge deck is shown in Fig. 2.

The underside of a typical grid used in this structure appears in Fig. 3.

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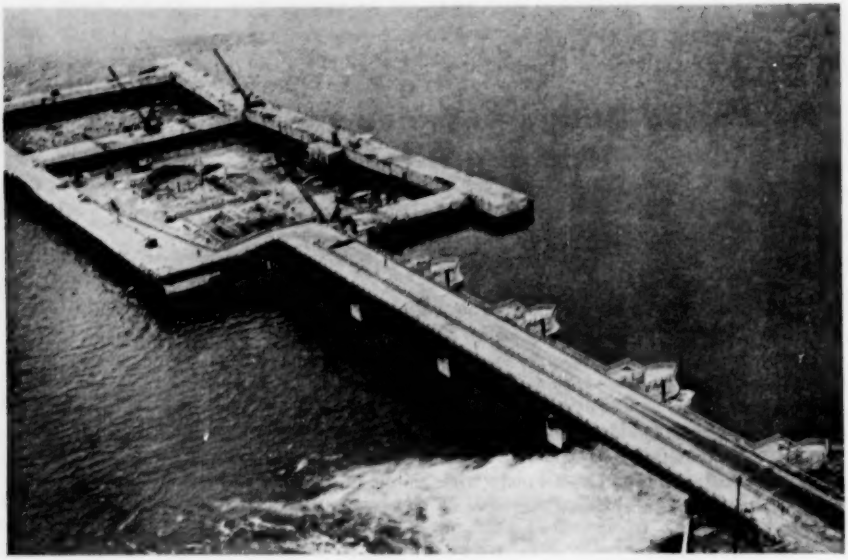

Fig. 1—Completed portion of bridge deck is on the right side of the long span shown here. The temporary construction bridge parallels it forming the left side of the divided roadway. Construction proceeds on another pier in the unwatered area

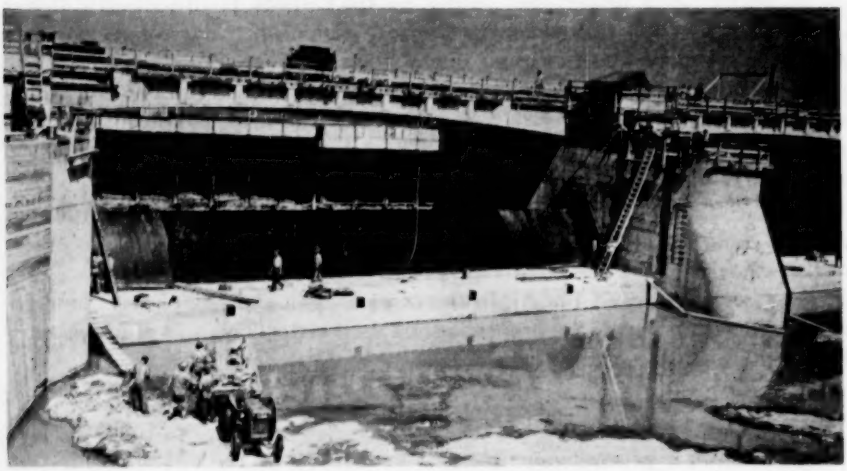

Fig. 2—Side view of bridge deck shows the 70-ft girder span in place between 15-ft members cantilevered from the piers

These grids, 70 ft long and 24 ft wide, are carried by 15-ft cantilevers projecting from the piers located between the gates.

Each grid consists of six longitudinal girders tied together by diaphragms with full moment connections. The transverse diaphragms are prestressed

so that all girders act simultaneously. To achieve maximum efficiency the beam spacing is varied so that the outer beams are closer together than the inner ones. An analysis was made of beam spacing to insure that maximum loading on any one beam would be approximately the same as that for any other beam. A 7-in. prestressed concrete slab spans between the girders acting compositely with the longitudinal girders.

Fig. 3—Under view of the girders. Note unequal spacing and diaphragms. Boxes between center girders are for ducts

At midspan the main girders are 42 in. deep with an area of 601 sq in., and each weighs 25.5 tons. They are stressed by means of eight prestressing bars of 1⅛-in. diameter. Beam ends are formed to correspond with beam seats at the end of the cantilevers, and the resulting shape of the end block created some difficulties in the detailing of anchorage layouts. Provision had to be made for adequate clearances for jacking equipment.

Because of the number of prestressing elements and transverse diaphragms, location of and clearance between longitudinal and transverse elements had to be carefully detailed.

The loading, span length, and girder spacing were such that the thickness of the deck slab became a problem. In composite prestressed concrete construction the weight of the deck slab is of considerable importance to design efficiency. The main beams alone usually have only half the strength of the composite section. The dead load effect of the deck itself is therefore considerably magnified unless the beams are propped during casting operations. Apart from the complications involved, it is difficult to control the stress distribution in the prestressed beams resulting from propping. On a span of this length a deck thickness of 7 in. can be considered critical. For thicknesses above this, little extra longitudinal bending strength is obtained, and the resulting decompression of the bottom flange of the beam at the time of casting of the deck becomes a serious problem. Consequently, the deck slab itself was prestressed to keep its dimensions within reasonable limits. The saving thus achieved in the beams outweighed the slight extra cost of the slab.

DESIGN CONSIDERATIONS

The bridge is subject to a load of a crane with boom at radius of 30 ft lifting a 6-ton concrete block. This produces a load of 48 tons on one track of the

crane. In addition the superimposed loading of trucks carrying 6-ton blocks had to be considered. It was expected that the cranes would normally operate inside the curbs of the structure, but it was necessary to provide for the eventuality that a crane might inadvertently climb the curb.

The distribution of load among the main longitudinal girders, coupled with the need of precise determination of resulting diaphragm stresses, constituted the major design problem.

Whether the structure is considered empirically or calculated precisely, the problem is to predict what in fact does occur in the completed structure. The degree to which the designer can be successful in his prediction is a measure of the progress he can make in developing new design types and in designing structures of maximum economy. Principal factors affecting load distribution among the girders are diaphragm stiffness, beam spacing, and torque rigidity of the system. The nature and thickness of the deck slab may have considerable effect on a bridge system without diaphragms, but where diaphragms are used the effect of the deck slab is chiefly confined to increasing diaphragm stiffness and in adding to the torque rigidity of the system. Its own transverse bending effect is negligible relative to that of the diaphragms.

Diaphragm stiffness and torque rigidity have fundamentally different effects. An increase or decrease of diaphragm stiffness will correspondingly increase or decrease primary stresses in the longitudinal members and diaphragms. No secondary stresses result. An increase in torque rigidity will tend to decrease primary stresses in heavily loaded members, increase stresses in lightly loaded members, and will result in secondary shear stresses particularly in the longitudinal members.

In a system with no transverse rigidity and no torque rigidity the beam directly under the load carries the whole load. In a system with infinite transverse stiffness, but no torsional stiffness, the center of gravity of reactions from the beams lies directly under the load, as shown in Fig. 4. Thus, if a load is at the center of the bridge deck all the beams will deflect uniformly. If the load is at the side of the bridge, there will be a uniform straight-line deflection distribution between the girders starting from a maximum at one side of the bridge to a minimum at the other end. There may actually be an upward reaction from the beams on the side opposite to the load.

In a system infinitely rigid in both transverse bending and torsion as shown in Fig. 5, no matter where the load is located all the beams deflect an equal amount.

In reality, of course, no system is entirely free of transverse bending stiffness or torque rigidity. True conditions are a mixture to a varying degree of both factors. It may be shown that the effect of longitudinal torque resistance is negligible in calculations involving widely spaced girders. It is, however, not negligible when girders are closely spaced or have exceptionally large torque moduli, or where beams are erected touching flange to flange.

While torque rigidity substantially increases the strength of certain types of structures, resulting secondary stresses may be of considerable importance.

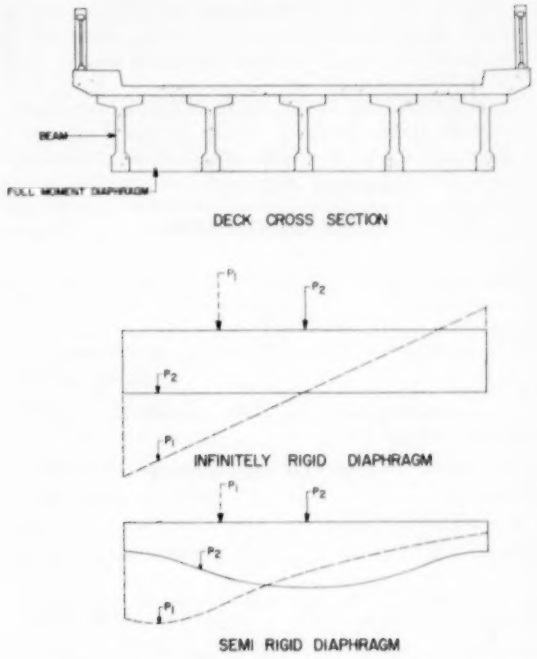

Fig. 4—Load distribution in a no-torsion system

Unless these secondary stresses are determined and unless we can be sure that creep and plastic flow resulting from such stresses will not in turn vary the load distribution pattern with time, we should be cautious in taking too much advantage of torsional rigidity. Torsional rigidity should be counted upon when all factors contributing to it in our assumptions, test specimens, or models are in fact actually present in the completed structure. This is a field in which additional research would be most welcome.

GRID ANALYSIS

A structure of this nature may be analyzed either by analogy or by a strict elastic analysis. Guyon has approached the problem by means of analogy to an anisotropic slab (having different bending properties in two different directions). Guyon does not consider torsional rigidity. Massonet has developed the Guyon method to take account of torsional rigidity. Both methods have been shown to be reliable for certain cases and for uniform moments of inertia.

In general, Europeans have recognized the importance of transverse strength in bridge structures and codes are either being prepared or have been prepared specifying minimum transverse strength.

None of the existing empirical methods of Guyon, Massonet, or others were applicable to the Niagara bridge due to the variable moment of inertia,

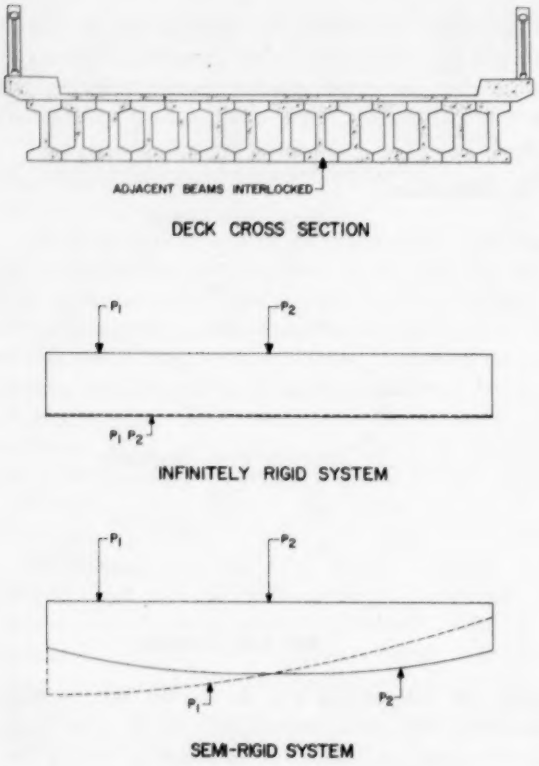

Fig. 5—Load distribution in a torsional system

the variable spacing, and the large number of diaphragms. Therefore, it was necessary to make an exact analysis of the grid. The method of analysis adopted was developed by Lazarides.*

For uniform or single-point loading the method of relaxation is the most applicable. The Dome of Discovery at the Festival of Britain was an outstanding example of the application of this method. For moving loads, influence fields must be calculated by the solution of the basic simultaneous elastic equations for the system. This was the method adopted for the solution of this problem.

The procedure consisted of the following steps:

1. Intersection point deflection calculations for the longitudinal and transverse beams considered separately under the action of unit point loads applied at each intersection point in turn.

2. Compounding elastic equations based on the principle of equating two distinct expressions for the difference of deflection of the same grid point. This produced 48 lineal elastic equations equal in number to the number of elastic unknowns which, therefore, made it possible to determine these unknowns.

*Lazarides, T. O., "The Design and Analysis of Openwork Prestressed Concrete Beam Grillages—Part I," *Civil Engineering and Public Works Review* (London), V. 47, 1952, pp. 471-473. See also Lazarides, T. O., *The Structural Analysis of the Dome of Discovery*, Crosby Lockwood and Sons, Ltd., London.

3. Reduction of the number of unknowns by breaking down the initial set of elastic equations into two sets with six unknowns and two sets with eight unknowns. This was done by the load transformation method and greatly reduced the work involved in solving the equations.

4. Solution by the straight Gauss method of each set of equations. To desensitize the equations, thus making manual solution practical, selected arbitrary constants were added to the system, to spread apart the individual inverse matrices. Work had to be carried out to 12 significant figures to insure a result accurate to four significant figures. After solution of the equations the arbitrary constants had to be compounded with the answers. The selection of the arbitrary constants calls for experience and a knowledge of the theory of equations.

5. Recompounding of the four sets of answers to produce the basic interaction forces at any point of the grid due to a load at any point.

6. Carrying of the stability and elastic checks to insure the validity and accuracy of the answers.

7. Drawing of the influence field for the grid.

8. Calculation of shears and bending moments for the loading considered.

In spite of numerous short cuts and systematic tabulation, five men worked for 6 weeks, 10 hr per day, to complete the original calculations. This effort was justified only by the large savings resulting from the internal load distribution in the 13-span structure. The method has since been coded for electronic computer, and the entire operation repeated in less than 2 hr at a cost below $300.

By hand, this analytical approach is of value only to special problems. The labor cost precludes it from use on smaller and less important structures. However, when carried out on a electronic computer, the method has a wide field of application.

CONSTRUCTION DETAILS

Prestressing system

The so-called "blank" design method has been much used or abused by designers wishing to insure competitive bidding on all phases of their work. The best that can be said for such procedures is that they have worked out not too badly in some cases. Specifying the center line of the prestressing force is an elementary exercise, but careful detailing is required for the practical location of actual prestressing elements and the housing of the anchorages. As long as simple beams are involved the problem is usually not great. When beams are continuous or when prestressing elements must be stopped or bent up, the original design may have to be altered substantially to accommodate certain systems. Otherwise, types of design not lending themselves to all systems cannot be considered. In the main, the effect of "blank" design is a general stultification of design.

To avoid this and yet to provide competition, bids were taken for the supply of prestressing materials and equipment. The Stressteel Co. was awarded the supply contract, and from that point forward, all details and dimensions were worked out based on this system. Several features, notably the end block details and the diaphragms, could not have been designed

without knowing exact element capacities, dimensions, permissible curvatures, and jacking clearances. Even the selection of the optimum girder shape would have been difficult without knowing what system was to be used.

Stirrups

Stirrups were detailed to provide support for the prestressing elements. They were made in halves so that the second half could be inserted after the element had been securely wired to the first half. Stirrups with multiple bends requiring great accuracy in bending and skill in placing were avoided.

Shear connectors

Composite construction requires an adequate shear joint between the beam and the deck. An important feature of a shear connection is its resistance to slip movements. In a grid structure where load transfers at intersection are apt to reverse from one point to the next, a rigid connector is of great importance.

In this case not only are the shear forces heavy, but conditions are such that a soft joint could disturb the stiffness relationships between the members. The most rigid joint possible is an actual shear key, but unreinforced keys cannot be relied upon when stresses are significant. Also, the deck must be held down onto the keys by steel projecting above the flange, making construction more complicated. If this steel has to supplement the shear capacity of the concrete key, the steel should be designed for the full capacity since there will be some movement before the steel can act and this movement may be sufficient to crack the concrete key.

In this bridge the shear forces both in the beam and at the joint are exceptionally heavy. There are two systems of shear connectors. The first is an extension of the stirrups, which means that for every pair of stirrups there are four bars through the joint. In addition, there are "concertina" bars in the top flange. These bars have a great advantage over straight dowels. They are easier to handle and to keep in place and they are twice as stiff as straight bars due to the effect of the loops.

Miscellaneous details

Holes for transverse bars are provided by pipe sleeves. The greater the beam spacing the more the tolerance that can be allowed in the location of these sleeves.

There is no point in saving materials by making shallow slopes on the tops of the bottom flanges. A steep slope may theoretically mean a few extra tons of prestress in a large beam, but practically, those extra tons are usually there anyway due to limitations imposed by available element capacities. A steep slope means assurance of good concrete in the bottom flange, a longer form life, greater ease in removing forms and a better surface finish. It is also easier to use dryer concrete and thus reduce cement costs and waste. The bottom flange is sturdier and is more capable of resisting impact forces. The loss of one beam would wipe out many times any saving resulting from reduction in concrete volume.

All inside and outside corners were chamfered, and there were no sharp changes of sections. This preserves forms and results in a stronger, better-looking product at little or no real extra cost.

Longitudinal mild steel is used in the form of #3 bars at the four corners of the I. This keeps the stirrups in line, makes a stronger cage which is less apt to become disturbed by concrete placement, and increases the cracking strength of the beam.

Stiffeners cast integrally with the beam were avoided where possible as these complicate formwork, reduce form life, and complicate the auxiliary mild steel. In this case the cast-in-place diaphragms stressed later provided the stiffener action needed. Only the edge girders have small projections on the outer side of the web to accommodate the transverse prestressing anchorages.

CONSTRUCTION PRACTICES

Materials

The concrete was of a minimum specified strength of 5000 psi. This work was carried out by the owner's forces and no trouble was encountered in producing concrete of this quality consistently.

All heats of prestressing bars were tested, and the stressing data were computed separately for each heat. The bars were found to be accurately cut, and careful planning of casting beds insured that all the beams were of exactly the right length to match the bars.

Casting operation

Casting operations were carried out in a yard close to the bridge site. Two beams were cast simultaneously, and four casting beds were provided.

While the main operations took place during the summer, some winter concreting was necessary. This was done by enclosing the casting beds in tarpaulins and running steam lines into the enclosure. Temperature was carefully controlled. The application of steam directly to pockets in the formwork under the beam was avoided as this practice has been found to be dangerous.

Forms

Careful attention was paid to the provision of gaps between panels to allow for expansion of the wood when wet and to prevent binding when forms were removed.

Traps were left at regular intervals in the top of bottom flange form to permit insertion of vibrations. Fig. 6 shows a section of the casting area with a girder being formed.

Stressing and handling

While the crews, previously inexperienced in this work, were at first somewhat apprehensive about this operation it was found that it quickly became

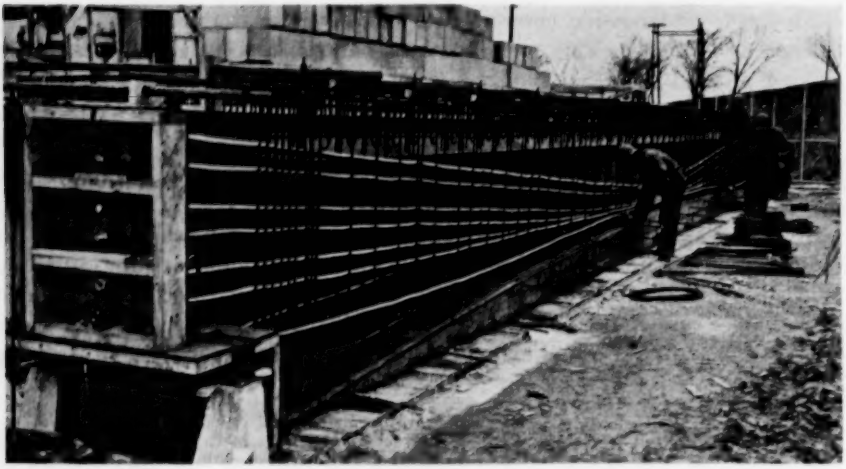

Fig. 6—Forming the girders. Note web reinforcement and shear dowels

routine. Stressing is always regarded as a major item by those unfamiliar with prestressing. In reality trouble is seldom encountered.

The beams were lifted by a specially designed cradle constructed from Bailey bridging, as shown in Fig. 7. Beams not required immediately were stored on racks in a yard close to the casting site. Beams were handled into place over a temporary construction bridge providing access to the cofferdams.

Decks and diaphragms

The deck and diaphragms were cast in place from formwork supported off the main beams. The formwork was then stripped and the transverse prestress applied. This operation proceeded smoothly. A view of the deck under construction may be seen in Fig. 8.

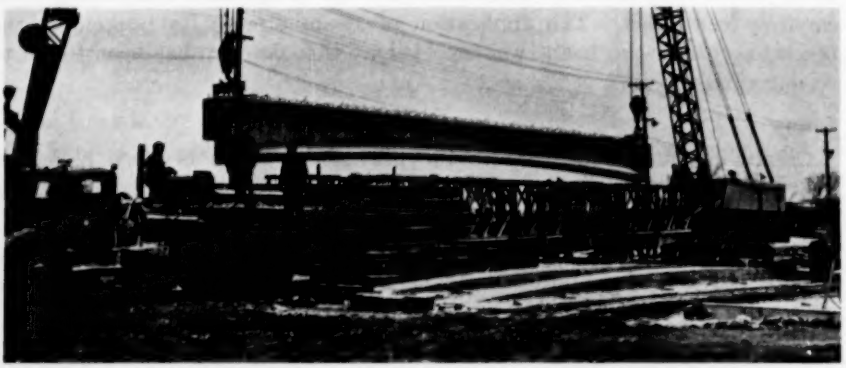

Fig. 7—Lifting girder. Holes are for diaphragm prestressing units. The girder will be transported on the Bailey bridging cradle

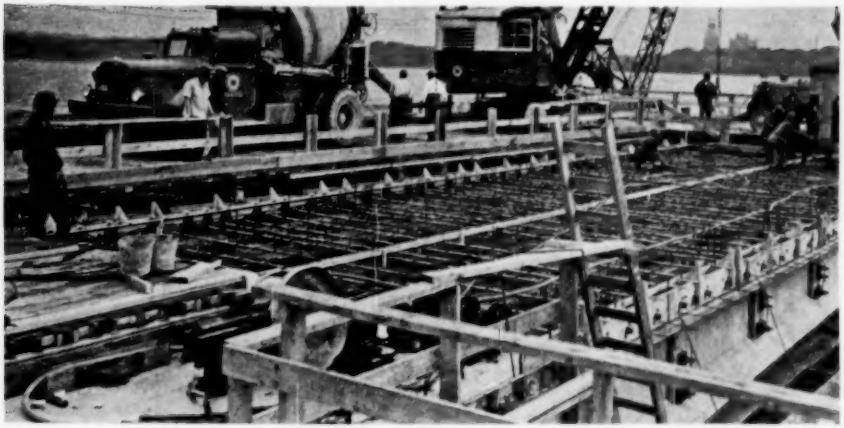

Fig. 8—Casting the deck. Note transverse prestressing elements

Grouting

To protect the prestressing elements and to obtain maximum ultimate capacity, the prestressing ducts were grouted at a pressure of 70 psi. It has been our experience that unless grout is applied under such pressure, voids may be left, permitting the accumulation of water next to the prestressing elements. While it remains to be proved that such stagnant water may actually harm the elements, it can do no good.

CONCLUSION

This job was ideally suited to prestressed concrete and was undertaken as a normal construction by a force of men with no previous experience in this method of construction. The results show that anyone with experience in the production of high-standard reinforced concrete can tackle with confidence a job in prestressed concrete.

The design problems encountered were of unusual interest, and their solution has led to the finalizing of a design method for bridge deck grids that is of general application. The preliminary results of load tests on completed portions of the structure have been encouraging and show that the method is probably the most accurate means yet developed of analyzing such a structure. The use of the electronic computer has made it possible to apply the method as a matter of routine to highway bridges. Economies thus achieved have been substantial. It is hoped that further developments will enable designers to take greater advantage of the torque resistance of grid systems when such will result in further economies.

For such discussion of this paper as may develop please see June 1957 JOURNAL.

Title No. 53-29

Particle Interference in Concrete Mixes*

By B. J. BUTCHER† and H. J. HOPKINS‡

SYNOPSIS

C. A. G. Weymouth's theory of particle interference postulates that fine particles infiltrate into the voids of larger particles. The authors' examination of Weymouth's method of calculation has shown his results to be in conflict with recognized physical laws. They advance a new method of applying the Weymouth criterion for particle interference in concrete mixes, based upon a consideration of the complete grading of solids instead of applying the criterion in turn to each successive size group.

INTRODUCTION

C. A. G. Weymouth's theory of particle interference, first presented in 1933, is based on the assumption that fine particles of an aggregate infiltrate into the voids of coarser particles. This seems a reasonable assumption since fine particles are more likely to be displaced by agitation or vibration of a fluid. It raises a problem for further study, however, of the fluidity necessary for the theory to apply.

Weymouth expressed the opinion that the extent of particle interference was governed by the relationship between the size of infiltrating particles and the minimum clearance between evenly spaced larger particles. He derived an expression for this minimum clearance, t, in the following form.

$$t = \left[\left(\frac{d_o}{d_a}\right)^{1/3} - 1\right] D_n \qquad (1)$$

Where

d_o = maximum bulk density or the absolute volume of 1 cu ft of fully compacted material of the size group being considered

d_a = relative density of that size group in the particular grading, or the absolute volume it occupies divided by the volume available for it after the addition of all larger particles

D_n = average size of particles being considered

DERIVATION OF WEYMOUTH FORMULA

Briefly, Eq. (1) was derived in the following way.

Let N be the number of particles in the size group per unit volume available.

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Then

$$d_a = NgD_n^3 \quad \ldots\ldots (2)$$

where g is a shape factor (assumed constant).

If these N particles were "blown up" to a size D_m so that they were in mutual contact in the densest packing, they would occupy a volume d_o, since maximum density depends upon shape but not upon size. Hence

$$d_o = NgD_m^3 \quad \ldots\ldots (3)$$

Eq. (2) and (3) give

$$\frac{D_m}{D_n} = \left(\frac{d_o}{d_a}\right)^{1/3} \quad \ldots\ldots (4)$$

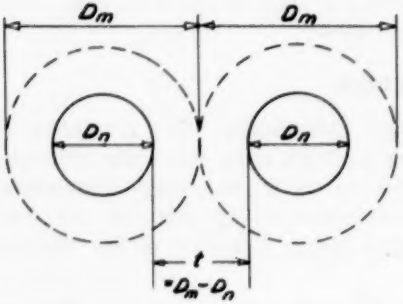

Fig. 1—Relationship of clearance to particle size

But from Fig. 1, $t = D_m - D_n$ so that

$$t = \left[\left(\frac{d_o}{d_a}\right)^{1/3} - 1\right] D_n \quad \ldots\ldots (1)$$

If the average diameter of the next smaller size group is D_{n+1}, then Weymouth stated that there is no particle interference if

$$D_{n+1} \leq \left[\left(\frac{d_o}{d_a}\right)^{1/3} - 1\right] D_n \quad \ldots\ldots (5)$$

APPLICABILITY OF WEYMOUTH FORMULA

The application of this formula to concrete aggregates is valid provided that the following conditions are satisfied:

1. Variations from spherical shapes introduce no significant error.
2. Variations in size throughout a size group may be allowed for by determining its average size.
3. The particles are mutually in contact after being "blown up."

Weymouth indicated that variations in shape would affect the value of d_o and thus no significant error would be introduced, and he implied that (2)

was true although he did not derive an expression for the average size. In applying the particle interference criterion to each successive size group, he assumed that condition (3) was satisfied. Now, although (1) and (2) may be reasonable, the applicability of (3) is open to doubt. If the largest size group is being considered, then these particles may be "blown up" to give mutual contact. Successively smaller size groups, however, are affected by the presence of all larger particles, and hence the extent to which a fine particle may be "blown up" should be determined *not* by the requirement that it will touch a particle of similar size, but by the fact that it will touch a particle of a larger size.

Inconsistencies

The authors, in investigating the application of Weymouth's theory, found that certain physical inconsistencies resulted from following the method which Weymouth adopted.

These inconsistencies are demonstrated in Appendix 1. They appear to spring from the general application of Eq. (5) to successive size groups, although that equation has only a particular application.

Fundamental application

As stated at the beginning, the theory of particle interference rests upon the assumption that *fine particles of an aggregate infiltrate into the voids of larger particles.* The authors are of the opinion that this requires *all* fine particles to infiltrate into the voids of *all* larger particles. Moreover, if a grading has minimum particle interference, then the minimum clear distance between all larger particles is not less than the effective size of all fine particles, and this requirement should hold regardless of what size is assumed in determining fine particles and larger ones.

Thus in Fig. 2, $x - x$ represents an arbitrarily chosen size, all particles

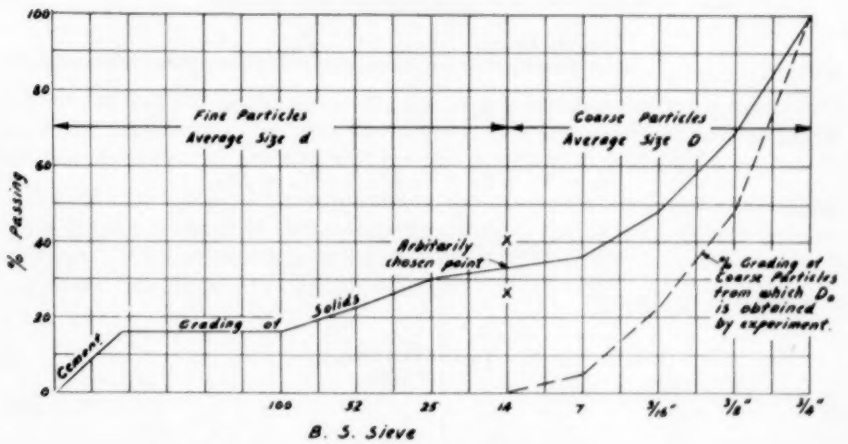

Fig. 2—Arbitrary definition of fine and coarse particles

coarser than $x - x$ have an average size D and all particles smaller than $x - x$ have an average size d.

Eq. (5) is completely valid for spheres of uniform size and it has been extended to apply to particles of varying sizes and approximately uniform shape. If the variation of sizes is not critical, then the equation can be applied to a range of sizes, provided that the average size of both coarse and fine fractions can be obtained. In this event, the requirement for minimum interference becomes

$$d \leqq \left[\left(\frac{d_o}{d_a}\right)^{1/3} - 1\right] D \quad \ldots\ldots (5a)$$

Where

d_o = maximum bulk density of all larger particles
d_a = relative density of all larger particles

Average size

Eq. (5) was derived by considering the volume of a specified number of particles and hence the average size of particles should be determined in the same manner.

The absolute volume of the larger particles

$$\left.\begin{aligned} &= d_a \\ &= NgD^3 \end{aligned}\right\} \quad \ldots\ldots (6)$$

Where

N = number of larger particles
g = Weymouth shape factor
D = mean diameter of all larger particles

Subdividing this fraction into a number of size groups

$$D_1, D_2, D_3 \ldots\ldots D_n$$

so that

$$N = N_1 + N_2 + N_3 + \ldots\ldots N_n = \Sigma N_1$$

Then

$$d_a = V_1 + V_2 + V_3 + \ldots\ldots V_n = \Sigma V_1$$

and

$$d_a = \Sigma N_1 g D_1^3 \quad \ldots\ldots (7)$$

From Eq. (6) and (7)

$$D^3 = \frac{g\Sigma N_1 D_1^3}{g\Sigma N_1} \quad \ldots\ldots (8)$$

But

$$N_1 = \frac{V_1}{gD_1^3}$$

so that

$$D^3 = \frac{\Sigma \dfrac{V_1 \; D_1^3}{D_1^3}}{\Sigma \dfrac{V_1}{D_1^3}} = \frac{\Sigma V_1}{\Sigma V_1 D_1^{-3}} = \frac{d_a}{\Sigma V_1 D_1^{-2}}$$

or

$$D = \left[\frac{d_a}{\Sigma V_1 D_1^{-3}}\right]^{1/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

The average diameter thus depends upon the grading.

Concrete gradings are usually specified in terms of sieves with openings in geometrical progression with a descending ratio of $1/2$. If the distribution of particles throughout one of these size groups is uniform (*i.e.*, it gives a straight line on the semi-log scale upon which gradings are plotted) and if this size group is subdivided into x $(\rightarrow \infty)$ subdivisions

$$D^3 = \frac{d_a}{\dfrac{V_1}{D_1^3} + \dfrac{V_2}{D_1^3(1/2)^{3/x}} + \dfrac{V_3}{D_1^3(1/2)^{6/x}} + \ldots\ldots + \dfrac{V_x}{1/8 D_1^3}}$$

$$= \frac{d_a \; D_1^3}{V_1(1 + 2^{3/x} + 2^{6/x} + \ldots\ldots 8)} \text{ since } V_1 = V_2 = V_3, \text{ etc.}$$

$$= \frac{x \; D_1^3}{(1 + 2^{3/x} + 2^{6/x} + \ldots\ldots 8)} \text{ since } d_a = x \; V_1$$

Hence

$$3.37 \; D^3 = D_1^3$$

or

$$D = 0.67 \; D_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Fundamental relationship for minimum particle interference

Thus for uniform grading in a size group, the average size is approximately $2/3$ of the maximum. Uniform distribution in a size group is assumed here, but any deviation in practice will involve a calculable correction.

Eq. (9) may be used for determining the average size of any grading and hence it may be used to determine the effective size of both coarse and fine fractions. With a notation in which capital letters refer to coarse particles and lower case letters to fine particles

$$D = \left(\frac{D_a}{\Sigma V_1 D_1^{-2}}\right)^{1/3}$$

and

$$i = \left[\left(\frac{D_a}{d_a}\right)^{1/3} - 1\right]\left[\frac{D_a}{\Sigma V_1 D_1^{-3}}\right]^{1/3} = \frac{D_a^{1/3} - d_a^{1/3}}{(\Sigma V_1 D_1^{-2})^{1/3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

If the total absolute volume of 1 cu ft of fully compacted solids conforming to the grading curve is V, then $V - D_a$ is the absolute volume of fine material.

Hence

$$d = \frac{(V - D_a)^{1/3}}{(\Sigma v_1 d_1^{-3})^{1/3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

So that the criterion that $d \leqq t$ for minimum particle interference is

$$\frac{(V - D_a)^{1/3}}{(\Sigma v_1 d_1^{-3})^{1/3}} \leqq \frac{D_o^{1/3} - D_a^{1/3}}{(\Sigma V_1 D_1^{-3})^{1/3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

It should be noted that D_o and D_a refer to the coarser particles combined in the proportions given by the grading curve.

Eq. (13) may be written in the form

$$\frac{V - D_a}{(D_o^{1/3} - D_a^{1/3})^3} \leqq \frac{\Sigma v_1 d_1^{-3}}{\Sigma V_1 D_1^{-3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13a)$$

But

$$\Sigma v_1 d_1^{-3} = \frac{V - D_a}{d^3} \text{ from Eq. (12)}$$

and

$$\Sigma V_1 D_1^{-3} = \frac{D_a}{D^3} \text{ from Eq. (9)}$$

Hence

$$\frac{V - D_a}{(D_o^{1/3} - D_a^{1/3})^3} \leqq \frac{(V - D_a)\, D^3}{D_a\, d^3}$$

$$\leqq \frac{V - D_a}{r^3\, D_a} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Where $r = d/D$ or the ratio of $\dfrac{\text{average size of fine particles}}{\text{average size of coarse particles}}$

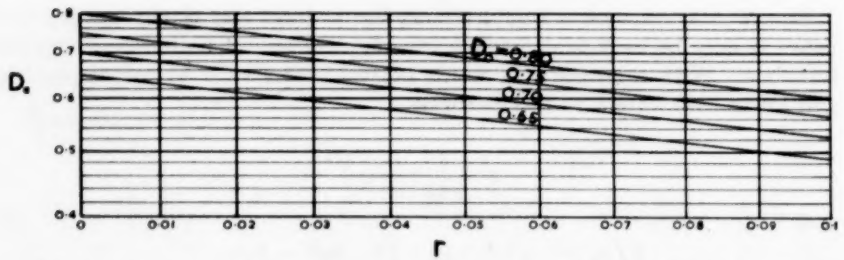

Fig. 3—Maximum volume of coarse material, D_a, without particle interference

Therefore

$$r^3 D_a \leqq (D_o^{1/3} - D_a^{1/3})^3$$
$$r D_a^{1/3} \leqq D_o^{1/3} - D_a^{1/3}$$

or

$$D_a \leqq \frac{D_o}{(1 + r)^3} \quad \ldots\ldots (15)$$

In Fig. 3, D_a is plotted against r for differing values of D_o for the limiting case of

$$D_a = \frac{D_o}{(1 + r)^3} \quad \ldots\ldots (15a)$$

This gives the maximum volume of coarse material, D_a, that can be introduced into a grading without causing particle interference.

EXAMINATION OF GRADING CURVES FOR PARTICLE INTERFERENCE

The examination of a particular grading curve for particle interference is shown in Table 1 to illustrate the following procedure.

Column 1—Sieve sizes

These include the sizes for the cement. As the general theory deals with the distribution of particle sizes irrespective of their function, it is clear that all solids should be included in the examination.

Column 2

The theory deals with relative sizes of particles, so that it is unnecessary

TABLE 1—EVALUATION OF GRADING CURVE FOR PARTICLE INTERFERENCE

1 B. S. Sieve	2 Relative size	3 Proportions by volume	4	5	6	7 D_o by test	8 D_a of grading	9 Maximum D_a, Fig. 3
¾–⅜-in.	16	39	$D^3 = 79$	$D^3 = 10.8$	$D^3 = 1.05$	0.65	0.340	
⅜–⅛-in.	8	17				0.67	0.487	
⅛-in.–No. 7	4	6	$D = 4.3$	$D = 2.21$		0.70	0.540	
No. 7–14	2	6			$D = 1.01$	0.74	0.591	0.72
No. 14–25	1	6	$d^3 = \frac{1}{14300}$			0.76	0.645	0.72
No. 25–52	½	9		$d^3 = \frac{1}{17600}$		0.79	0.722	0.715
No. 52–100	¼	3	$d = \frac{1}{24.2}$		$d^3 = \frac{1}{27000}$	0.80	0.749	
Cement	$\frac{1}{17}$	$\frac{14}{100}$		$d = \frac{1}{26.0}$	$d = \frac{1}{30}$	0.87		
$\frac{d}{D} = \hat{r}$ (fraction)			$\frac{1}{104}$	$\frac{1}{57.5}$	$\frac{1}{30.3}$			
r (decimal)			0.0096	0.0174	0.0330			

to insert the actual particle sizes. The relative values given in this column have been chosen to give convenient values of d^3 and D^3.

Column 3

These are proportions by volume, and again are relative values.

Column 4

An arbitrary division is made at the British No. 14 sieve size. All particles passing this sieve are assumed to be fine material.

For the coarse material

$$D^3 = \frac{39 + 17 + 6 + 6}{\frac{39}{16^3} + \frac{17}{8^3} + \frac{6}{4^3} + \frac{6}{2^3}} = 79$$

and for the fine material

$$d^3 = \frac{6 + 9 + 3 + 14}{6 + 9\times2^3 + 3\times4^3 + 14\times32^3} = \frac{1}{14{,}300}$$

It will be apparent that only the latter terms of the denominator are significant, and this allows a rapid calculation.

Column 5

The arbitrary division is here made at the No. 25 sieve, so that all particles passing this sieve are regarded as fine material.

Column 6

The arbitrary division is made at No. 52 sieve.

Column 7

This gives the maximum absolute volume of the coarse material combined in the proportions of the grading curve. Thus the first figure (0.65) represents the maximum bulk density of ¾ to ⅜-in. aggregate. The second figure (0.67) is the maximum bulk density of ¾ to ⅜-in. and ⅜ to 3/16-in. particles combined in the proportions 39:17, respectively. The third figure (0.70) is the maximum bulk density of ¾ to ⅜-in, ⅜ to 3/16-in. and 3/16-in. to No. 7 particles combined in the proportions 39:17:6, respectively, and so on.

Column 8

This gives the absolute volume of all material coarser than the last named sieve. Thus the first figure is (39/100) × 0.87, and the second figure in the column is (39 + 17)/100 × 0.87 and so on.

Column 9

This gives the maximum volume of coarse material that can be introduced without causing particle interference. It is read off the graph of Fig. 3. Thus for the arbitrary division at the No. 14 sieve, D_c from column 7 is 0.74, and $r = 1/104 = 0.0096$. From Fig. 3, maximum $D_a = 0.72$. The volume of

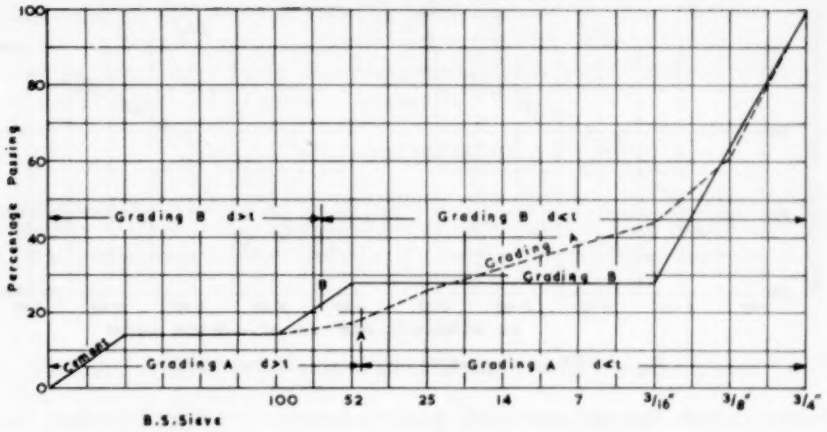

Fig. 4—A comparison of gradings for particle interference

coarse material included in this grading (column 8) is 0.591. Hence there is no particle interference at this size. Particle interference begins at a size between the No. 25 and 52 sieve.

INTERPRETATION OF RESULTS

$$D_o = \frac{D_o}{(1+r)^3} \qquad (15a)$$

gives the maximum volume of coarse aggregate that can be introduced into a mix without causing particle interference. The grading analyzed in Table 2 is plotted in Fig. 4 as grading A, and a gap-graded aggregate (B) is also shown. Grading A relies upon an increasing value of D_o to displace the point of particle interference to the left, while a low value of r achieves the same object for grading B. The latter method gives a grading with more fines, so

TABLE 2—CALCULATION OF t

Groups	0 to No. 100	No. 100 to No. 48	No. 48 to No. 28	No. 28 to No. 14	No. 14 to No. 8	No. 8 to No. 4	No. 4 to ⅜ in.	⅜ in. to ¾ in.	¾ in. to 1½ in.
Avg D, mm	0.084	0.220	0.441	0.877	1.764	3.54	7.08	14.1	28.5
Sand, percent	1	6	31	24	19	16	3		
Gravel, percent							25	50	25
1. Absolute volume	0.003	0.020	0.102	0.079	0.062	0.053	0.105	0.190	0.095
2. Relative spaces	0.294	0.314	0.416	0.495	0.557	0.610	0.715	0.905	1.000
3. d_s	0.010	0.064	0.246	0.160	0.111	0.087	0.147	0.210	0.095
4. d_o	0.600	0.630	0.650	0.650	0.650	0.650	0.650	0.650	0.650
5. t, mm	0.245	0.251	0.161	0.523	1.41	3.38	4.55	6.48	25.6
6. Critical t			0.161	0.523				6.48	
7. D_t, mm			0.220	0.441				7.08	
An alteration in percentage of gravel to ⟶ alters only the two columns on far right.							25	0	75
1. Absolute volume							0.105	0	0.285
2. Relative spaces							0.715	0.715	1.000
3. d_s							0.147	0	0.285
4. d_o							0.650	0.650	0.650
5. t, mm							4.55		9.00

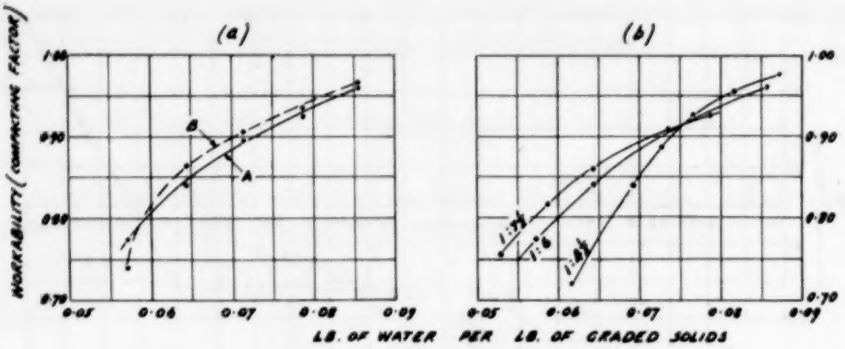

Fig. 5—Effect of particle interference on workability

that although the size at which particle interference begins is finer than for grading A, the volume causing interference is greater. The relative importance of these two factors is difficult to assess, for at low water contents the behavior of a mix is affected by other variables.

It would appear from wrokability tests upon these two gradings that the size at which particle interference begins is the fundamental criterion. Fig. 5(a) shows the compacting factors for gradings A and B plotted against weight of water per pound of solids. Grading B becomes more workable than A when the mix has sufficient fluidity for the Weymouth criterion to apply. This is further borne out by Fig. 5(b) which gives similar curves for mixes containing aggregate conforming to grading A and cement in the proportions 4½:1, 6:1, 7½:1 (by weight), respectively, to give combined gradings as shown in Fig. 6.

As previously stated, the problem of the fluidity necessary for the theory to apply warrants further study, but it appears that the Weymouth criterion

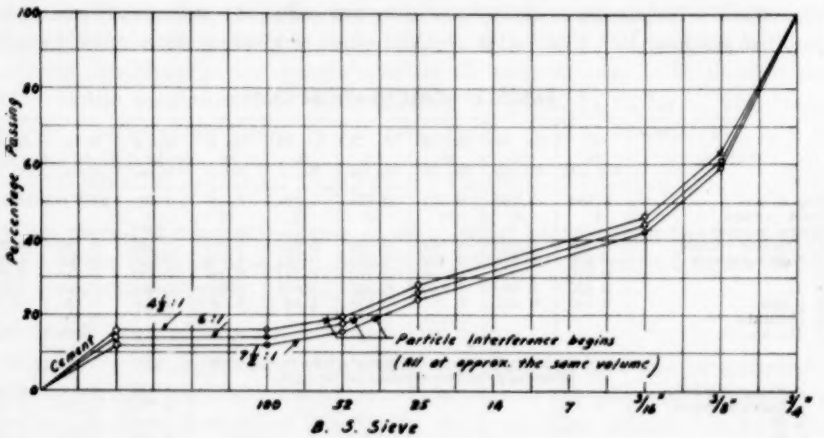

Fig. 6—Variation of grading of solids with cement content

is applicable in the range of hand tamped concrete, whereas vibrated concrete lies outside its range. The criterion for this latter method of compaction is more likely to be minimum surface area, provided that sufficient fines are incorporated to prevent segregation.

ACKNOWLEDGMENT

The experimental work for this paper was carried out at Canterbury University College, Christchurch, New Zealand, by the first named author[3] under the supervision of the second, who is Head of the Department of Civil Engineering.

REFERENCES

1. Weymouth, C. A. G., "Effect of Particle Interference in Mortars and Concrete," *Rock Products*, V. 36, No. 2, 1933, pp. 26-30.

2. Dunagan, W. M., "The Application of Some of the Newer Concepts to the Design of Concrete Mixes," ACI JOURNAL, June 1940, *Proc.* V. 36, pp. 649-684.

3. Butcher, B. J., "Study of Grading of Aggregates and the Workability of Concrete," unpublished thesis, Canterbury University College, New Zealand, 1953.

APPENDIX 1—INCONSISTENCIES IN WEYMOUTH METHOD

Dunagan[2] has pointed out that the Weymouth formula

$$t = \left[\left(\frac{d_o}{d_a}\right)^{1/3} - 1\right] D_n \quad \ldots\ldots (1)$$

rationalized the exponential

$$p = \left(\frac{d}{D}\right)^x \quad \ldots\ldots (16)$$

Where

p = proportion (by volume) of material finer than d
d = size of particles being considered
D = size of the largest particles
x = an arbitrary exponent

Dunagan's paper included a proof by T. C. Powers that the curves of Eq. (1) and (16) are identical.

In the Weymouth formula, if d_a is constant for all fractions,

$$p = (1 - d_a)^n \quad \ldots\ldots (17)$$

where p has the same meaning as in Eq. (16) and n = number of size group counting the largest size group as 0.

If the two curves are identical, then equating Eq. (16) and (17)

$$\left(\frac{d}{D}\right)^x = (1 - d_a)^n \quad \ldots\ldots (18)$$

then

$$\left(\frac{d}{D}\right) = r^n$$

where r is the descending ratio of size groups, and substituting in Eq. (18)

$$r^{nx} = (1 - d_a)^n$$

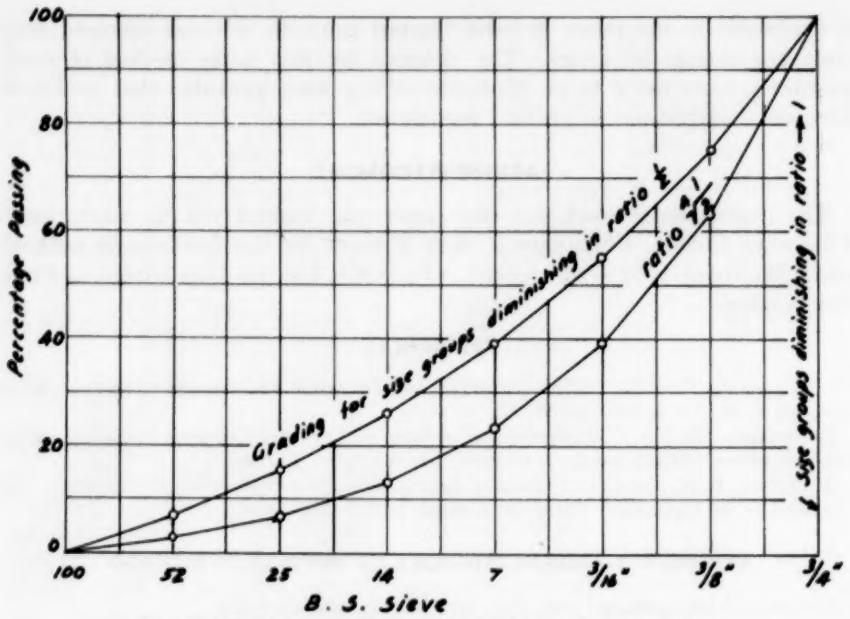

Fig. 7—Variation of Weymouth grading with sieve series

so that

$$x = \frac{\log (1 - d_a)}{\log r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Thus x depends on d_a and r and not on d_a alone. If x is dependent only upon the aggregate, $1 - d_a$ must vary directly with r. Fig. 7 shows the gradings obtained when r is $\frac{1}{2}$, $1/\sqrt[4]{2}$, and when $r \rightarrow 1$. On the basis of calculation used by Weymouth, minimum particle interference therefore depends not only on the physical properties of the aggregate, but upon an arbitrary decision taken in calculating it. This appears to be in conflict with recognized physical laws.

The application to a specific grading curve of the Weymouth criterion leads to the conclusion that the grading of fines is dependent upon the volume but not the grading of coarse particles. This will be clear from an examination of Table 2 which reproduces part of Table I of Weymouth's original paper.[1]

Further, in his Table I, Weymouth gave t for No. 100 to No. 48 as 0.251 despite the fact that for the next larger size t is 0.161. These points indicate that the method has no "memory." It is therefore reasonable to conclude that the method adopted by Weymouth is in error and it would appear that the error lies in applying the criterion in turn to a series of arbitrarily chosen size groups.

For such discussion of this paper as may develop please see 1957 JOURNAL pages.

CURRENT REVIEWS

of Significant Contributions in Foreign and Domestic Publications

Dams

Experimental and mathematical analysis of arch dams, with special reference to Dokan

DERYCK N. DE G. ALLEN, LETITIA CHITTY, ALFRED J. S. PIPPARD, and ROY T. SEVERN, *Proceedings*, Institution of Civil Engineers (London), Part I, V. 5, No. 3, May, 1956, pp. 198-258 (including discussion)
Reviewed by ARON L. MIRSKY

A combination of analytical and experimental methods was used in designing Dokan dam in Iraq. Due to lack of analysts versed in the trial-load method, relaxation was used to solve the exact elastic equations, which were formulated in terms of three-dimensional displacements rather than stresses. A simplified solution (the "tumbler" solution), based on the assumption dam was a section of a complete cylindrical tube with any vertical section being similar to the maximum dam section, was also obtained; in this case problem reduced to one of plane (two-dimensional) stress. Experimental studies were carried out on cast-rubber models using water loading. Analytical and experimental results are given and compared; agreement was quite close except in the case of vertical displacements; major discrepancies observed are discussed and accounted for.

Details of the rubber models and their preparation are given, as is an analysis of the effect of the difference in Poisson's ratio and a method of correcting for it. The relaxation results are outlined in a lengthy (pp. 230-244) appendix.

Some interesting editorial comment on this paper appears in *The Engineer* (London), V. 201, No. 5218, Jan. 27, 1956, pp. 117-118. Additional discussion is printed in the *Proceedings*, Part I, V. 5, No. 4, July 1956, pp. 412-434.

Design

Analysis of shells of translation (Berechnung von Translationsschalen)

W. ZERNA, *Oesterreichisches Ingenieur-Archiv* (Vienna), V. 7, No. 3, 1953, pp. 181-187
Reviewed by GUNHARD ÆSTIUS ORAVAS

Author develops the membrane and deformation theories for shells of translation with positive Gaussian curvature. The membrane theory yields a differential equation in terms of stress resultant acting in the surface of the shell. The state of strain and the boundary perturbation are expressed by differential equations in terms of normal displacements. The basic theory is all developed in tensor notation, which allows an elegant and concise presentation of the subject matter.

Analysis of a frame with three bays

V. A. MORGAN, *Concrete and Constructional Engineering* (London), V. 51, No. 8, Aug. 1956, pp. 451-459

A continuation of a series by this author. The series has included the analysis of one-bay frames in July, 1955, and two-bay frames in February, 1956 ("Current Reviews," ACI JOURNAL, Aug. 1956, *Proc.* V. 53, p. 235). Illustrates the application of the moment distribution method to a gable frame with three or more bays for vertical or lateral loads and for fixed or hinged bases.

A part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 5, Nov. 1956, *Proceedings* V. 53. Address 18263 W. McNichols Rd., Detroit 19, Mich. Where the English title only is given in a review, the book or article reviewed is in English. If it is followed by a foreign title the work reviewed is in that language. In those cases where the foreign title cannot conveniently be set in type or is not available, the language of the original article is indicated in parentheses following the English title. Copies of articles or books reviewed are **not** available through ACI. Available addresses of publishers are listed in the June "Current Reviews" each year. In most cases ACI can furnish addresses of publications added later.

For those members that cut apart this section for pasting on cards for card indexes, a limited number of complimentary reprints of the "Current Reviews" section are available from ACI headquarters on request.

Effect of depth of beam upon the modulus of rupture of plain concrete

C. P. Linder and J. C. Sprague, *Proceedings*, ASTM, V. 55, 1955, pp. 1062-1084 (including discussion)

The load at failure of plain concrete beams does not vary as the square of the depth, and the modulus of rupture (as conventionally calculated) varies inversely with the depth. An attempt is made to rationalize this behavior. A stress distribution, partly parabolic in the tensile zone, is offered as an explanation and as a source of an equation to predict more closely results of tests. The paper concludes with correction factors for conversion to an equivalent basis for modulus of rupture tests on different depth beams.

Moment redistribution in statically indeterminate prestressed concrete structures beyond the elastic phase (in Dutch)

B. Visser, *Cement* (Amsterdam), V. 7, No. 11-12, Dec. 1955, pp. 279-281

Reviewed by John W. T. Van Erp

Increasing use of indeterminate prestressed structures leads to refinement in design methods. In steel structures the formation of yield hinges makes the redistribution of moments fairly simple and in good agreement with behavior of the actual structure. Prestressed concrete is structurally a much more complicated material and its behavior under the above circumstances is not quite known and understood. Only continued research will help solve the problem. The article enumerates all the test programs and papers which were read at the International Federation of Prestressed Concrete conference in Amsterdam, 1955. Final conclusions as summarized by Guyon seem to indicate that a complete redistribution takes place at ultimate load, in other words the formation of plastic hinges until the structures becomes unstable and fails.

Vibrations of systems with several degrees of freedom (Schwingungen von Systemen mit mehreren Freiheitsgraden)

August Nesitka, *Der Bauingenieur* (Berlin), V. 30, No. 7, July 1955, pp. 244-248

Reviewed by Aron L. Mirsky

Mathematical discussion of free and forced vibration of four, two, and three degrees of freedom. As a numerical example, a frame-type foundation is investigated.

Materials

Effect of calcium chloride on corrosion of steel reinforcement in concrete (in Russian)

L. I. Panfilova and G. A. Kozlova, *Beton i Zhelezbeton* (Moscow), No. 1, 1955, pp. 33-34. *Chemical Abstracts*, V. 50, No. 6, Mar. 25, 1956, p. 4478

Highway Research Abstracts May 1956

Experiments were made with reinforced concrete made from portland cement of different mineral composition and containing 3 percent $CaCl_2$ by weight of the cement. The steel showed little corrosion; after 28 days in water and air, corrosion started at the ends of the rods. In most cases, corrosion did not increase as the concrete hardened. Addition of 2 percent $CaCl_2$ is permissible since it causes practically no corrosion. Addition of 18 percent $CaCl_2$ + 5 percent NaCl, and 10 percent $CaCl_2$ + 5 percent NaCl by weight of the water, to cold concrete caused strong corrosion after 28 days in water and air; corrosion increased with time.

Hardening processes of cements during treatment with high-pressure steam (in Russian)

P. I. Bozhenov and G. F. Suvorova, *Tsement*, V. 21, No. 5, 1955, pp. 4-8

Ceramic Abstracts May 1956 (Kamieh)

At temperatures up to 1000 C in saturated steam, the hydration products have characteristics different from those of normal hardening. As pressure rises, the ratio of crystalline products to gel-like products increases. Strength tests show that for each cement, depending on its mineral composition, there is an optimum condition of autoclaving within the limits of 15 to 25 atmospheres which will give the most rational ratio of crystalline and colloidal hydration products.

Cold twisted reinforcing steel (in Dutch)

P. J. Van Tussenbroek, *Cement* (Amsterdam), V. 8, No. 13-14, Feb. 1956, p. 332

Reviewed by John W. T. Van Erp

Some rules for the use of high-strength steel are given, based on maximum permissible crack widths that will still prevent corrosion. These rules have been set up by the building supervision departments of the biggest cities. For exterior exposures a width of 0.008 in. and an allowable unit

stress of 24,000 psi, for interior exposure a width of 0.012 in. and a stress of 30,000 psi s given. The minimum reinforcing is 0.2 percent for statically indeterminate and 0.3 percent for statically determinate structures based on effective depth. If diagonal tension reinforcement is necessary at least 75 percent of such reinforcement shall be bent up bars, and the remainder stirrups. Compressive reinforcement may not be taken into account. A chart gives the reinforcement percentage for certain crack widths, steel stresses, and moments.

Importance of free lime in portland cement for the resistance of concrete to attack (in German)

F. W. Meier-Grolman, *Zement-Kalk-Gips* (Wiesbaden), V. 9, 1956: No. 1, pp. 15-28; No. 2, pp. 58-71
Ceramic Abstracts
July 1956 (Hartenheim)

The unbound lime hydrate in the set cement is composed of the so-called free lime and of the lime hydrate separated in the course of the hydraulic hardening of the cement from the lime-rich Ca silicates of which only a small part is used in the formation of lime-rich aluminates. Results of tests to evaluate the role of free CaO in concrete are summarized as follows: (1) The hydrolytically separated Ca hydrate is not attacked as it is surrounded by the gel mass; (2) The critical limiting value for the content of free CaO is 0.5 percent of the cement for concrete mixes containing up to 300 kg per cu m of cement. Mixes with over 400 kg per cu m of cement are less susceptible owing to the protective effect of their density. (3) The C_3A content should not exceed 6 percent. (4) The best method for estimating the resistance of concrete is the determination of the modulus of elasticity by the acoustic method. (5) On the basis of these tests and of theoretical considerations, the attack modulus, A, was formulated as the quotient of the topochemically reacting constituents of portland cement and the constituents reacting in the liquid phase: $A = (C_2S + C_3S + C_4AF)/(\text{free CaO} + C_3A + K_2O + CaSO_4)$; this permits the determination at an early stage of the resistance of concrete on the basis of composition. The attack of concrete decreases with higher A and higher density. Contains 102 references.

Precast Concrete

Reinforced concrete buildings with prefabricated external wall panels (in Danish)

C. S. Forum and K. E. Buurgaard Jensen, *Beton Teknik* (Copenhagen), V. 22, No. 1, 1956, pp. 1-11
Reviewed by T. Hermansen

Describes the construction of six multistory reinforced concrete buildings with prefabricated external wall panels. About 2000 units were fabricated. The size of each panel was 9¼ x 11½ ft, the weight about 2.3 tons. They were cast with a dense concrete facing and a lightweight concrete backing. Concrete molds were used.

Properties of Concrete

Method for determining the moisture condition of hardened concrete in terms of relative humidity

C. A. Menzel, *Proceedings*, ASTM, V. 55, 1955, pp. 1085-1109 (including discussion)

Describes a rapid method and simple apparatus for determining the moisture condition of hardened concrete block and expressing the results directly in terms of relative humidity. The method gives reliable indications of the moisture condition of concrete block in about 15 to 30 min and should be particularly useful as an acceptance test and also as a routine plant control test. The results are particularly useful in that they indicate the average humidity in service to which the block is suited or whether further drying is required. With some modification the method can also be used to determine moisture condition of hardened concrete at the surface or in the interior of large specimens.

Structural Research

Results of experiments on beams prestressed by deformed bars

A. D. Ross, *Civil Engineering and Public Works Review* (London), V. 50, No. 588, June 1955, pp. 639-642
Highway Research Abstracts
Apr. 1956

A report on a series of tests made to examine the use of deformed bars for pre-

tensioning and anchorless post-tensioning and to assess the performance of the resulting beams, 10 ft 6 in. long. Provided the grouting is efficient, anchorless post-tensioning with deformed bars is a practicable form of construction; there was no end-slip over the whole range of loading up to failure. Pretensioned beams made with deformed bars have a better load-factor and a lower crack-spacing than a comparable post-tensioned beam with conventional end anchors.

General

Dynamics in tall structures, especially in industrial buildings (Dynamik im Hochbau, besonders bei Industriebauten)

J. Geiger, *Der Bauingenieur* (Berlin), V. 30, No. 6, June 1955, pp. 210-217

Reviewed by Aron L. Mirsky

A thorough discussion of vibrations in a textile mill, including resonant frequencies and forces on walls (solid and brick), and upper-story floor slabs supporting a number of machines. Also discussed are the vibrations of the structure as a whole, characteristic frequencies and modes, vibration measurements, and methods of reducing or damping out vibrations.

Paper was abstracted (by author) in *Zeitschrift VDI*, V. 96, No. 36, Dec. 21, 1954, pp. 1230-1232.

Lectures at the yearly convention of the German Concrete Association (in German)

Deutscher Beton Verein E. V., Wiesbaden, 1955, 368 pp.

Reviewed by John W. T. Van Erp

All thirteen lectures are reported, the first of which goes far beyond the scope of reinforced concrete. It treats the newer concepts about the origin of the universe, the origin and age of all inorganic matter, the chemical elements. Deductions about earthly matter from celestial observations are explained, thus giving the yearbook a much wider aspect than routine concrete problems. Somewhat related to this, however, is the subject of the next lecture: the durability of concrete and deterioration. Other subjects: design by the *n*-free method; ultimate strength method of design; concrete dams; concrete reconstruction in the port of Hamburg; successful cooperation of structural engineer (concrete designer) and architect, as in the design of the market hall in Hamburg. Also treated are determination of water content in aggregates, tests of joist floors under long-term sustained loading, practical and theoretical problems of prestressed concrete, elimination of the uneven stress distribution in prestress cables due to friction, and behavior of overloaded prestressed concrete. The yearbook, lives up to its tradition of giving an up-to-date picture of our best knowledge of reinforced concrete and the solution of its many problems.

Elevated storage tank in Bagnoli (Depósito elevado, en Bagnoli)

Riccardo Morandi, *Informes de la Construccion* (Madrid), No. 78, Feb. 1956, Paper No. 581-11, 4 pp.

A 132,000-gal. reinforced concrete industrial water supply tank is supported at 115 ft (35 m) by a reinforced concrete tower. The top section of the tower contains a 20,000 gal. portable water supply tank. Reinforcing details and some design data are given.

Some churches with shell-type roof construction (Einige Kirchen mit Dachausbildungen in Schalenform)

Kurt Hirschfield, *Der Bauingenieur* (Berlin), V. 30, No. 5, May 1955, pp. 161-171

Reviewed by Aron L. Mirsky

Describes two churches containing some interesting structural elements. St. Sebastian's Church, Aachen, has a large cylindrical shell (barrel vault) roof, complicated by having two unequal spans longitudinally, a short shell entering normal to the longitudinal axis, and a non-rigid support along one edge; a corrugated-type roof; a reinforced concrete open-web girder acting as the edge support of the main shell roof; a diagonal-grid room covering; and a circular footing on elastic foundation. The Church of Christ the King, Neuss, has a doubly-curved thin shell cupola with open edge beams. Architectural treatment and stress analysis are both emphasized in article.